1 Introduction

In spite of an increasing interest in marginal costs for pavement renewal, reliable estimates of these costs are not easily found. An international review shows that there is an urgent need for new results based on reliable data sources (Bruzelius, 2004). The most utilized approach in the few studies that are available is the estimation of cost functions on cost and traffic data. Using some flexible function, the relation between traffic and costs can be estimated without many assumptions regarding the underlying process.

An alternative method, more heavily founded in a theoretical understanding of road deterioration and investment strategies, has also been proposed. Assume that a deviation from the desired pavement quality level is accepted before a pavement renewal is undertaken. Such a policy can be explained by lumpy adjustment costs that make continuous adjustments inefficient. Instead a discrete series of repavements is generated. The repavements are separated by pavement lifetimes that are negatively related to traffic intensity. The strength of this relation is measured by the deterioration elasticity. A marginal increase in traffic intensity shortens the pavement lifetime and advances the series of repavements. As a consequence the present value for repavement costs increases, which is interpreted as the marginal cost. The higher the deterioration elasticity (in absolute value terms), the higher is the marginal cost.

The so-called fundamental theorem is based on this model. Assuming no traffic growth, that road deterioration is unaffected by climate conditions
and that road damage is proportional to cumulative standard axes\textsuperscript{1}, i.e. the deterioration elasticity is $-1$, Newbery (1988) finds that the marginal cost is equal to the average cost. The validity of the fundamental theorem can however be questioned. Empirical studies have shown that the fraction of costs allocated to vehicles is 60-80 percent in hot dry climates and somewhat lower, 20-60 percent in freezing temperature climates (Newbery, 1990). Also, a study of the relation between traffic intensity and the occurrence of cracks in the pavement shows that the pavement lifetime is shortened by 0.1-0.8 percent when heavy vehicle traffic increases by one percent, indicating that the marginal cost is 10-80 percent of the average cost (Lindberg, 2002). These numbers result from a weaker relation between traffic and pavement lifetime than the fundamental theorem implies, probably due to climate effects.

We will use a similar model in this study, but add some features for greater realism. We drop the assumption that road damage is proportional to cumulative axes. Instead a pavement lifetime function is estimated, from which deterioration elasticities for heavy goods vehicles and passenger cars can be derived. We use a large data set covering all pavement renewals on the Swedish national road network since the 1950s. The variable of main interest is the time that elapses between two repavement occasions, i.e. the pavement lifetime. It is found that both passenger cars and heavy goods vehicles deteriorate the road, a result that contradicts the usual assumption that deterioration is caused by heavy vehicles (standard axes) only. The deterioration elasticity of heavy goods vehicles is found to be low (in a absolute value sense), indicating the lifetime of road pavement to be less sensitive to traffic than what is implied by the fundamental theorem. The deterioration elasticity of passenger cars is low as well. Consequently the marginal costs are also low.

The paper is organized in the following fashion. Next we introduce the reader to a model showing the relation between traffic and pavement deterioration starting with a structural model over the factors affecting road deterioration and pavement renewal. Then the relation between traffic and renewal costs is described. Here we draw on earlier research to which some new features are added in order to increase realism. After a description of the data set the empirical method is presented. The reader is also introduced to

\textsuperscript{1}The definition of a standard axis starts from the so-called fourth power law of the relation between axle weight and road wear. The number of standard axes associated with a specific vehicle is the sum of the fourth power of each axis weight divided by $10^4$ (DOT, 1994).
duration analysis, the technique used to estimate the relation between traffic and pavement lifetime. After estimation of the pavement lifetime function and derivation of the deterioration elasticities we are finally able to compute marginal costs for pavement renewal using unit prices for pavement work.

2 Pavement deterioration and renewal

The renewal of road pavement is triggered by some pavement quality condition. In theoretic work, this condition is generally stated in terms of road roughness. Pavement renewal is undertaken when roughness reaches some critical level. For a road with certain non-traffic characteristics (strength), one usually assumes roughness to be a function of the wear and tear of cumulative standard axes. This approach effectively rules out any potential impact from passenger cars on road deterioration. Although this assumption seems to be widely accepted the reality is more complex. According to the Swedish Road Administration’s (SRA) official maintenance repair policy, the criterion for pavement renewal is a combination of rutting and roughness conditions (SRA, 1990). Including rutting, which is a consequence mainly of passenger cars using studded tyres, as a “trigger” for pavement renewal, means that the pavement lifetime cannot be assumed to be a function of heavy goods vehicles (HGV) only, but of passenger cars as well.

Our understanding of road deterioration and pavement renewal is based on information from SRA and is concluded in a directed acyclic graph (figure 1) where arrows indicate (assumed) causation. The figure shows that one reason for repavement is road deterioration. Road deterioration in turn is a function of original design, construction quality, traffic volume, road geometry, pavement age and climate conditions (Zarghampour, 2005). Deterioration is not the sole determinant of repavement however. How far the deterioration process is allowed to proceed before repavement is undertaken is determined by maintenance class specific policies (SRA, 1990).

\[^2\]The average number of standard axes on heavy goods vehicles in Sweden is 1.3, while on passenger cars it is practically zero due to their limited weight.

\[^3\]An example: the highest standard in Sweden is that average rut depth should be less than 17 mm measured over a 400 m section and less than 20 mm over a 20 m section. The corresponding IRI levels (international roughness index) are 2.5 and 4.0 mm/m respectively. For a discussion about ways to express the pavement condition see also Ekdahl (2000).

\[^4\]For a thorough presentation about direct acyclic graphs and causality, see Pearl (2000).
Figure 1: Deterioration model (HGV:s=heavy goods vehicles)
The road is worn down by traffic, a process expressed by a deterioration function in average traffic flow, \( Q \) and time, \( t \). \( \bar{Q}_t \) is the average traffic flow, for the first pavement lifetime, observed at \( \tau \leq T \).

Abstracting for the moment from some factors in figure 1, we assume that the actual deterioration level depends on cumulative traffic on the road since its last repavement and on time. Also, although deterioration is a vector consisting of both rutting and roughness, we use a single number/index to express its level. As a last simplification we use a single measure for all
traffic. Since cumulative traffic is the product of average traffic and time, deterioration can be expressed as \( D = D(Q, t) \). In figure 2 we have plotted deterioration as a function of average traffic and time, where the time dimension is depicted by the x-axis. At calender time 0 deterioration is at its minimum level, \( D_{\text{min}} \), and reaches the critical level, \( D_{\text{max}} \) at \( T \). Then the road is given a new pavement and the process starts over again.

Now consider an additional vehicle at time \( \tau \), \( \delta Q \). This vehicle adds some extra deterioration to the road and shifts the solid line upwards. As a consequence \( D_{\text{max}} \) is reached earlier and the first pavement lifetime is shortened by \( \delta T \). This effect corresponds to the one resulting from an increase in the average traffic flow, \( \delta Q \), as illustrated by the dashed line in figure 2. After \( \tau \) traffic falls back to a constant level so the following re-pavement intervals keep their original length \( (T) \) but are advanced.

Let us now specify the deterioration function as:

\[
D(t) = D_{\text{min}}e^{Q^\beta t}
\]

The pavement lifetime \( T \) is then defined by:

\[
D_{\text{min}}e^{Q^\beta T} = D_{\text{max}}
\]

which is equivalent to:

\[
T = \ln D_{\text{max}} - \ln D_{\text{min}} \equiv \frac{\alpha}{Q^3}
\]

Take logs:

\[
\ln T = \ln \alpha - Q^3 \ln Q
\]

For homogenous roads the pavement lifetime function above is valid, but heterogeneity complicates matters somewhat. A type of heterogeneity that is simple to understand is different critical levels for different roads, i.e. various \( D_{\text{max}} \) lines. This would generate dispersion in \( T \) for roads with equal traffic intensity and all other factors as well. Another type of heterogeneity is introduced if “road strength” in some sense is different for different roads.

In the following the relation between traffic volume and pavement lifetime is analysed, using survival analysis. The central concepts of this area are given in for instance Kiefer (1988) and Lancaster (1990) and are repeated here for convenience of the reader.
Let $\mu$ be the expected lifetime of a road pavement, $\mu = E(T)$. Let $F(t)$, $S(t)$ and $\lambda(t)$ denote the cumulative distribution function, the survivor function and the hazard function respectively, with definitions given below. The cumulative distribution function expresses the probability that the lifetime is shorter than $t$.

$$F(t) = P(T < t)$$  \hspace{1cm} (5)

The probability that a road will last at least as long as $t$ is called the survivor function:

$$S(t) = P(T \geq t) = 1 - F(t)$$  \hspace{1cm} (6)

Given that a road pavement has lasted until $t$, the hazard, i.e. the rate for failure at $t$ is:

$$\lambda(t) = \frac{f(t)}{S(t)}$$  \hspace{1cm} (7)

3 The theoretical framework for pavement renewal

Pavement renewal is here assumed to be undertaken at a constant unit cost, $C$ (SEK/sqm). With $r$ being the discount rate, the present value cost at $T$ for all future repavements is:

$$PV C_T = C(1 + e^{-rT} + e^{-r2T} + \ldots + e^{-rnT})$$  \hspace{1cm} (8)

The limit for this value when the number of future repavements approaches infinity is:

$$\lim_{n \to \infty} PV C_T = C \frac{1}{(1 - e^{-rT})}$$  \hspace{1cm} (9)

Relating back to the preceding section we now consider a scenario where the average traffic flow during the first pavement lifetime is $\overline{Q}_T$ and the time is $\tau < T$, that is $T$ is not yet observed. From the above we know that, after $T$, pavement lifetimes will be fixed, $\overline{T}$, forever. Note therefore, that at this

\footnote{SEK is the Swedish currency unit.}
time the series of pavement lifetimes consists of two distinct intervals: (i) the interval that will end $T - \tau$ years ahead and (ii) all intervals thereafter. The present value cost for all future repavement works at $\tau$ is:

$$PV C_{\tau} = C \frac{1}{(1 - e^{-r\tau})} e^{-rv}$$

(10)

where $v \equiv T - \tau$. With traffic volume having influence on $T$ it obviously affects $PV C_{\tau}$ as well. The marginal present value cost associated with additional traffic is caused by the fact that the present pavement lifetime, $T$, is shortened, advancing the whole series of future repavings. As an effect, all pavement expenses get closer in time, and $PV C_{\tau}$ increases. The marginal cost for structural pavement repair at time $\tau$ is the increase in $PV C_{\tau}$ associated with a traffic flow increase at that time.

$$MC_{\tau} = \frac{\delta PV C}{\delta Q} = \frac{\delta PV C_{\tau}}{\delta T} \frac{\delta T}{\delta Q} = -C_r \frac{e^{-rv}}{(1 - e^{-rT})} \frac{\delta v}{\delta Q} \frac{\delta T}{\delta T}$$

(11)

In order to make the interpretation useful (in the empirical analysis we establish a relation between $T$ and average traffic) we would like to rewrite the expression in equation 11 in terms of changes in $Q_I$. Following Lindberg (2004) we define the deterioration elasticity as:

$$\varepsilon = \frac{\delta T}{\delta Q_I} \frac{\overline{Q_I}}{T}$$

(12)

The deterioration elasticity is a measure of the responsiveness in pavement lifetime to a change in average traffic intensity. If $Q_I$ increases one percent the percentage change in $T$ is $\varepsilon$. The relation between a momentary traffic change relevant for marginal costs and deterioration elasticity is:

$$\frac{\delta T}{\delta Q_I} = \frac{\delta T}{\delta Q_I} \frac{\delta Q_I}{\delta Q_I} = \left[ \frac{\delta Q_I}{\delta Q_T} \approx \frac{1}{T} \right] = \frac{\varepsilon}{Q_I}$$

(13)

The interpretation of the approximation\(^6\) is that a small shift in traffic intensity at time $\tau$ leads to a shift in the average traffic volume over the whole period equal to $\frac{1}{T}$. Now rewrite equation 11 using equation 13:

\(^6\)It can be shown that the error of the approximation depends on the difference between the terminal traffic volume, $Q_T$ and $\overline{Q_I}$. In case of equality, the approximation is exact (see appendix). If one assumes that the marginal cost is caused by a temporary shock on an otherwise stable traffic flow, the approximation is thus justified.
\[ MC_\tau = \frac{\delta PVC}{\delta Q_\tau} = \frac{\delta PVC_\tau}{\delta T} \frac{\delta T}{\delta Q_\tau} = -Cr \frac{e^{-ru}}{(1-e^{-rT})} \frac{\varepsilon}{Q_I} \] (14)

The average MC over all possible remaining lifetimes is the expected marginal cost taken over a probability density function of \( \nu, g(\nu) \). In general the expected value of marginal costs over all remaining lifetimes:

\[ E[\frac{\delta PVC}{\delta Q_\tau}] = -\varepsilon r \frac{C}{Q_I} \int_0^\infty \frac{e^{-ru}}{(1-e^{-rT})} g(\nu) d\nu \] (15)

The integration limits are given by the lowest and highest possible ages of a road.

In equation 15 we see that the expected marginal PVC depends on the unit cost of pavement work, \( C \), the interest rate, \( r \), the mean traffic volume, \( Q_I \), and the distribution of remaining lifetimes. The probability density function of \( \nu \) is (Lancaster, 1990):

\[ g(\nu) = \frac{S(\nu)}{\mu}, 0 < \nu < \infty \] (16)

In earlier research, \( T \) has been assumed to be deterministic, that is \( f(t) \) is assumed to be degenerate with all the probability mass concentrated at \( \mu \). This follows from an assumption under which the pavement deteriorates deterministically with traffic, and that the lifetime of a road pavement comes to its end exactly when its quality falls to a pre-determined level. Under that condition \( g(\nu) \) is uniform (Lancaster, 1990), that is:

\[ g(\nu) = \frac{1}{T}, 0 < \nu < T \] (17)

Substitution of the uniform distribution into 15 leads to a very simple form for expected marginal PVC (equation 19) (Lindberg, 2004).

\[ E[\frac{\delta PVC}{\delta Q_\tau}] = -\varepsilon r \frac{C}{Q_I T} \frac{1}{(1-e^{-rT})} \int_0^T -\frac{1}{r} e^{-ru} du = -\varepsilon r \frac{C}{Q_I T} \frac{(1-e^{-rT})}{(1-e^{-rT})} \] (18)

Assuming further that all future re-pavement intervals are of the same length as the present, i.e. \( \bar{T} = T \), we have:

\[ E[\frac{\delta PVC}{\delta Q_\tau}] = -\varepsilon \frac{C}{Q_I T} \] (19)
Thus, under the assumption of the pavement lifetime $T$ being deterministic, the expected marginal present value cost is computed as the deterioration elasticity times the average cost $\frac{C}{Q_i T}$. Thus, in these terms the fundamental theorem, which states that $MC = AC$ implies that $\varepsilon = -1$.

### 3.1 Empirical modelling

In the empirical analysis for we assume $T$ to follow the flexible and simple Weibull distribution with parameters $\gamma > 0$ and $\alpha > 0$. The value of $\alpha$ determines the properties of the hazard rate. With $\alpha < 1$ the hazard decreases with time while $\alpha = 1$ implies constant hazard. Higher value means increasing hazard with $\alpha = 2$ being the breaking point for hazard functions that increase less than or more than proportionally to time. The distribution function, survival function and hazard function of the Weibull distribution are:

\[
F(t) = 1 - e^{-\gamma t^\alpha} \quad (20)
\]

\[
S(t) = e^{-\gamma t^\alpha} \quad (21)
\]

\[
\lambda(t) = \gamma \alpha t^{\alpha - 1} \quad (22)
\]

Then, using the general form of elapsed duration pdf (16) and the survival function of a Weibull distribution (21), we have the following pdf for remaining lifetimes:

\[
g(\upsilon) = e^{-\gamma \upsilon^\alpha} \mu, \quad 0 < \upsilon < \infty \quad (23)
\]

Substitute this Weibull-related remaining lifetimes pdf into 15:

\[
E\left[\frac{\delta PVC}{\delta Q_\tau}\right] = -\varepsilon \frac{C}{\mu Q_i (1 - e^{-r T})} \int_0^\infty e^{-r \upsilon - \gamma \upsilon^\alpha} d\upsilon \quad (24)
\]

No analytic solution to the integral in 24 is available since no primitive function can be found. Instead, numerical integration, e.g. Simpson quadrature, can be utilized.

The first factor is the deterioration elasticity. The second factor, $\frac{C}{\mu Q_i}$, is simply an average cost for each vehicle over the pavement lifetime. The
rest of equation 24 is due to the construction of this marginal cost from a present value function and its expected value over a non-uniform distribution of different road ages. Comparing 24 to 19 we see that the case where \( T \) is assumed to be deterministic (and thus \( \tau \), the elapsed duration, have a uniform distribution) can be used as a useful benchmark where this factor is unity.

4 Data

We use a large database containing observations of every completed renewal interval in the Swedish national road network between 1928 and 2005. Every time a road section is renewed, a new record with elapsed time since the last repavement and other information describing the section is added to the database. Technically, the observations make up a “flow sample”. The data set also includes variables describing the flow of passenger cars and HGV:s (both expressed as annual average daily traffic, AADT). Additionally the database contains variables specifying road width, speed, road type and to which region of SRA the road belongs. We choose from the database a subset of observations including records for the measures hot mix (asphalt), semi hot mix (e.g. oil gravel and soft bitumen), surface dressing B (gravel on bitumen) and fictive (where road standard assessment IRI reveals that renewals/improvements have been done, but no specific measure is recorded.) With this selection all roads with some kind of “hard” surface are included while gravel roads are excluded together with preparatory measures. This subset consists of 119,137 observations (no missing values in width, speed or traffic variables), where the earliest starting point for a renewal interval is 1951. It is worth noting that the data is censored. For each road with its last pavement lifetime ending before 2005, there exists another interval that has not yet ended, i.e. it is censored. The number of censored observations is 35,809.

Table 1 contains some descriptive statistics for our data set. The average lifetime of a road, i.e. the pavement lifetime is about 12 years with a variation between 3 and 46 years. An average road section is passed by 3364 passenger cars and 268 HGV:s per day. The minimum and maximum values for both vehicle types are outliers. The 99:th percentile for HGV:s is 1990 and for the flow of passenger cars it is 26,010. The average speed limit on the

\[7] The database has been compiled by SRA and has been kindly provided by Johan Lang.
Table 1: Descriptive statistics (N=119,137)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pavement lifetime (T)</td>
<td>12.119</td>
<td>7.558</td>
<td>3</td>
<td>46</td>
</tr>
<tr>
<td>AADT hgv ($Q_h$)</td>
<td>268.334</td>
<td>622.493</td>
<td>1</td>
<td>91,800</td>
</tr>
<tr>
<td>AADT pass. cars ($Q_p$)</td>
<td>3,364.266</td>
<td>5,425.502</td>
<td>1</td>
<td>67,200</td>
</tr>
<tr>
<td>Speed (Km/h)</td>
<td>76.067</td>
<td>18.029</td>
<td>20</td>
<td>110</td>
</tr>
<tr>
<td>Road width (m)</td>
<td>7.637</td>
<td>2.561</td>
<td>1.9</td>
<td>2.7</td>
</tr>
</tbody>
</table>

**Traffic class ($Q_p$)**

<table>
<thead>
<tr>
<th>Class</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 1,999</td>
<td>0.607</td>
<td>0.488</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2,000 – 3,999</td>
<td>0.143</td>
<td>0.351</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4,000 – 7,999</td>
<td>0.127</td>
<td>0.333</td>
<td>0</td>
<td>1</td>
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<tr>
<td>≥ 8,000</td>
<td>0.121</td>
<td>0.326</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**SRA region**

<table>
<thead>
<tr>
<th>Region</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern</td>
<td>0.165</td>
<td>0.371</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Central</td>
<td>0.171</td>
<td>0.376</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Stockholm</td>
<td>0.078</td>
<td>0.268</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Western</td>
<td>0.238</td>
<td>0.426</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Mälardalen</td>
<td>0.136</td>
<td>0.343</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>South-eastern</td>
<td>0.142</td>
<td>0.349</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Skåne</td>
<td>0.068</td>
<td>0.252</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Road type**

<table>
<thead>
<tr>
<th>Type</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>European highway</td>
<td>0.123</td>
<td>0.329</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>National road</td>
<td>0.211</td>
<td>0.408</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Primary county road</td>
<td>0.195</td>
<td>0.396</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Secondary county road</td>
<td>0.293</td>
<td>0.455</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Tertiary county road</td>
<td>0.174</td>
<td>0.379</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Sec/Ter county road</td>
<td>0.001</td>
<td>0.033</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 2: Average pavement lifetime per SRA region

<table>
<thead>
<tr>
<th>Pavement lifetime ($T$)</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years</td>
<td></td>
</tr>
<tr>
<td>Northern</td>
<td>14.15</td>
</tr>
<tr>
<td>Central</td>
<td>12.88</td>
</tr>
<tr>
<td>Stockholm</td>
<td>8.87</td>
</tr>
<tr>
<td>Western</td>
<td>11.56</td>
</tr>
<tr>
<td>Mälardalen</td>
<td>11.30</td>
</tr>
<tr>
<td>South-eastern</td>
<td>11.55</td>
</tr>
<tr>
<td>Skåne</td>
<td>13.80</td>
</tr>
<tr>
<td>All regions</td>
<td>12.12</td>
</tr>
</tbody>
</table>

Road sections in our data is 76 kilometres per hour, with observations spread between 20 km/h and 110 km/h. The average road width is 7.6 metres. The three lowest sections of the table contain dummy variables indicating traffic class, SRA region and road type. The dominating traffic class is the one containing road sections with a flow of 0-1,999 passenger cars per day. Of all road sections, 61 percent belong to this traffic class. The rest is quite evenly distributed among the three higher traffic classes. The largest share of pavement renewals is recorded in the western region of SRA, while Skåne has the lowest share. The different shares in the regions is probably due to differences in region size and the possibility that the length of reported renewals varies between regions. About 70 percent of the records concern secondary or tertiary county roads. The rest of the data set consists mainly of primary county roads (21 percent) and European highways (12 percent).

Computing regional pavement lifetime averages, we see that the time elapsing between two pavement occasions varies heavily between the regions. In table 2 we show the pavement lifetime in years. To see the differences and to enable comparison with later analyses, we have also computed an index based on the regional averages, where Skåne has an index of 1. The most startling feature in the table is the low values observed in the Stockholm region, where the average pavement lifetime is below nine years, compared with a national average of more than twelve years. This results in an index value of just 0.643. The longest pavement lifetimes are found in the Northern region and in Skåne, with averages of 14.15 and 13.80 years respectively. The central region has a pavement lifetime average slightly below thirteen. The Western, Mälardalen and South-eastern regions have average pavement
lifetime between eleven and twelve years.

5 Econometric modelling

In order to compute the expected marginal present value cost we need estimates of the deterioration elasticity, ε, and the Weibull parameters α and γ. As before $\bar{Q}_t$ is the average traffic volume and we also have a vector of covariates $N$, including a constant. $\beta_Q$ and $\beta_N$ are the respective coefficients. Consider now the linear model (the empirical counterpart to equation 4):

$$-\alpha \ln T = \ln \bar{Q}_t \beta_{Q} + N' \beta_N + u \quad (25)$$

With $u$ being a random error following an extreme value distribution, this is the accelerated duration specification of a Weibull regression model. With this specification we have (Kiefer, 1988):

$$\gamma = \exp(\ln \bar{Q}_t \beta_Q + N' \beta_N) \quad (26)$$

and the hazard:

$$\lambda(t) = \exp(\ln \bar{Q}_t \beta_Q + N' \beta_N) \alpha t^{\alpha - 1} = \bar{Q}^\beta_Q \exp(N' \beta_N) \alpha t^{\alpha - 1} \quad (27)$$

$\bar{Q}^\beta_Q$ measures the effect of traffic on the deterioration. It is then of interest to notice that $\alpha$ measures the time/weather effect. Hence a value of $\alpha > 2$ indicates that the pavement deteriorates from simply weather exposure as time passes by.

Having estimates of $\beta_Q$ and $\alpha$ an estimate of the deterioration elasticity can be computed:

$$\hat{\varepsilon} = \frac{\delta \ln T}{\delta \ln Q} = -\frac{\hat{\beta}_Q}{\hat{\alpha}} \quad (28)$$

The variance of this estimate is estimated by applying the delta method (see for instance Greene, 1997) to equation 28, using the variances and covariance of the parameter estimates $\sigma^2_{\hat{\alpha}}$, $\sigma^2_{\hat{\beta}_Q}$ and $\sigma_{\hat{\alpha} \hat{\beta}_Q}$.

$$\sigma^2_{\hat{\varepsilon}} = \alpha^2 \sigma^2_{\hat{\beta}_Q} + \hat{\beta}_Q^2 \sigma^2_{\hat{\alpha}} + 2\alpha \beta \sigma_{\hat{\alpha} \hat{\beta}_Q} \quad (29)$$

$N$ should include all factors, besides traffic, that have an impact on the pavement lifetime and without which $\beta_Q$ cannot be estimated consistently. Therefore, based on the structure of figure 1, we include in $N$ therefore, road
width, maximum allowed speed, geographical region and road categories. A larger road width implies that traffic can be distributed over a larger space, which should give longer pavement lifetimes. Speed might affect the deterioration caused by each vehicle, region will contain an approximate climate factor and road category is important since it determines the status of a road in the transportation system and is a crucial factor of the structural repair policy (SRA, 1990). The official road typology used is (SRA, 2002): The road is part of an international main road network for Europe.

European highway: The road is part of an international main road network for Europe.
National road: The road is part of a network that has been designated by the government as especially important for the national welfare.
Primary county road: Road of national interest.
Secondary county road: Road of general regional interest. Our data also contain Tertiary county roads.

5.1 Estimation

The analysed data is a censored “flow sample” that can be analysed with well established methods (see for instance ?). Our coefficients are estimated by maximum likelihood and the covariance matrix derived from the Hessian.

5.1.1 Results

The estimated models have a good overall fit as indicated by the LR-tests (p-values). The reduced model is included for assessment of model stability. With one exception all coefficients in the complete model are significant. All estimated coefficients in table 3 have been divided by \( \hat{\alpha} \) to enable easier interpretation. As a consequence the coefficients for log traffic flows (passenger cars and HGV:s) are the deterioration elasticities. In the complete model the deterioration elasticity for HGV:s is -0.04 (95 percent confidence interval C.I [-0.049 -0.031])\(^8\). Thus, an additional percent of HGV:s means that the pavement lifetime decreases by 0.04 percent, quite a small number. For passenger cars the corresponding number is -0.052 (95 percent C.I [-0.061 -0.043]). Even if the deterioration elasticity of passenger cars is small, it is significant and also larger than elasticity of HGV:s. Consequently one must question the usual assumption that pavement lifetime is independent of passenger car traffic. The results also tell us that the pavement on wider roads

\(^8\)This is a very low value. As discussed above, the fundamental theorem implies an elasticity of -1. Earlier estimations indicates an elasticity between -0.1 and -0.8, depending on road base strength (measured by surface curvature index, SCI) (Lindberg, 2003).
Table 3: Results from accelerated duration model. All estimates have been divided by $\hat{\alpha}$ and can be interpreted as ordinary regression coefficients. The dependent variable is lnT (log of pavement lifetime).*

<table>
<thead>
<tr>
<th></th>
<th>Reduced model</th>
<th>Complete model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>est</td>
<td>t</td>
</tr>
<tr>
<td>ln Passenger cars</td>
<td>-0.026</td>
<td>-6.02</td>
</tr>
<tr>
<td>ln HGV:s</td>
<td>-0.067</td>
<td>-17.25</td>
</tr>
<tr>
<td>ln Speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln Width</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northern</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stockholm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>West</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mälardalen</td>
<td>-0.006</td>
<td>-0.68</td>
</tr>
<tr>
<td>South-eastern</td>
<td>-0.070</td>
<td>-8.51</td>
</tr>
<tr>
<td>European highway</td>
<td>-0.337</td>
<td>-34.18</td>
</tr>
<tr>
<td>National road</td>
<td>-0.118</td>
<td>-15.17</td>
</tr>
<tr>
<td>Primary county road</td>
<td>-0.191</td>
<td>-22.86</td>
</tr>
<tr>
<td>Secondary county road</td>
<td>-0.169</td>
<td>-19.96</td>
</tr>
<tr>
<td>Constant</td>
<td>3.164</td>
<td>202.27</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.898</td>
<td>395.25</td>
</tr>
<tr>
<td>lnL</td>
<td>-90,270</td>
<td></td>
</tr>
<tr>
<td>LR</td>
<td>4.413</td>
<td></td>
</tr>
<tr>
<td>p</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>119,137</td>
<td></td>
</tr>
</tbody>
</table>

* Skåne is the reference region dummy. The road type dummies should be compared to the excluded tertiary road dummy.
lasts longer and that higher speed limits cause the pavement lifetime to decrease. A comparison between the complete and the reduced models shows that the estimated deterioration elasticities are pretty stable for different model specifications.

The Weibull-parameter (of the complete model), $\alpha$, is 1.94, which is found not to be higher than 2. ($H_0 : \alpha > 2$, t-value=-12.13). We can thus reject the hypothesis that there is a time or weather effect.

Looking at the coefficients of the regional dummies we observe geographical differences in the pavement lifetimes. From equation 25 we see that the dummy variables can be interpreted as multiplicative factors on the pavement lifetime. We can thus interpret the exponential of these dummies as a kind of pavement lifetime index, where Skåne, the reference region, has an index of 1. This index is shown in figure 3 where we see that the shortest pavement lifetimes are found in the Northern region, followed by, in order, Central, South-eastern, Stockholm, Mälardalen, Skåne and Western. Compared to the average pavement lifetimes of each region in table 2, it is obvious that the conditioned analysis performed here gives another picture. The most striking example is the Northern region, which, unconditionally on traffic and other regressors, had the longest pavement lifetimes, but is now shown to have the shortest. Also, Stockholm, where the pavement lifetime seemed very short, is less extreme in a conditional analysis.

5.1.2 Computation of marginal costs

From the regression estimates we derive estimates of the deterioration elasticities of passenger cars and HGV:s, $\hat{\varepsilon}$ and the Weibull-parameters $\hat{\alpha}$ and $\hat{\gamma}$. The deterioration elasticities and $\hat{\alpha}$ are constants and thus common to all observations. On the other hand, $\hat{\gamma}$ is a function of our regressors (see

\[
T = \exp\left(\frac{lnQ_i ' \beta_{Q_i} + N' \beta_N + u}{-\hat{\alpha}}\right)
\]

Now, let $\alpha_r$ be a regional dummy coefficient and $N^*$ all other covariates. Then $\alpha_r \cup N^* = N$. Rewrite the expression above:

\[
T = \exp\left(\frac{lnQ_i ' \beta_{Q_i} + N^* ' \beta_{N^*} + \alpha_r + u}{-\hat{\alpha}}\right) = \exp\left(\frac{\alpha_r}{-\hat{\alpha}}\right) \exp\left(\frac{lnQ_i ' \beta_{Q_i} + N^* ' \beta_{N^*} + u}{-\hat{\alpha}}\right)
\]
Figure 3: The regions of the Swedish Road Administration. The pavement lifetime index is shown within parentheses.
equation 26) and will thus take on different values for different observations.

The unit cost for pavement work, \( C \) (SEK/sqm) is available from road authorities or road contractors. It varies for roads with different traffic intensity as well as for various regions. Also, it varies with respect to the number of earlier repavement occasions. The engineering assessment (Lindberg, 2004, see) of this cost is in the range 10-160 SEK/sqm, with an average of 65 SEK/sqm\(^{10}\). In the following we will use this average, as an example, to compute marginal and average costs.

The estimated parameters and unit cost information facilitate the computation of equation 24. The integral is evaluated with the Simpson quadrature. We then get the estimated marginal cost:

\[
E\left[ \frac{\delta PV C}{\delta Q_i} \right] = -\hat{\varepsilon} C \times 1000 \times w \times r \times 365 \hat{\mu} \hat{Q}_I \hat{r} (1 - e^{-rT}) \int_0^\infty e^{-r\upsilon - \hat{\gamma}_\upsilon \hat{\alpha}} d\tau \quad (30)
\]

which can be factorized for easier interpretation. First, we have the negative of the deterioration elasticity. Next, we have a fraction which is the average cost per vehicle during a pavement lifetime. The unit pavement cost is multiplied by the square meters of pavement on one kilometer of road (1000 \( \times \) \( w \)). The expected pavement lifetime, \( \hat{\mu} \), is expressed in years and therefore multiplied by 365, the annual number of days. Last, we have a factor caused by our choice of using a Weibull distribution for pavement lifetimes. If pavement lifetimes are deterministic, we have seen that this factor is equal to one. Using our data, and assuming that all future pavement lifetimes have the same length as the first, \( (\hat{T} = T) \) and a discount rate of 0.04\(^{11}\), this factor is estimated to be 0.83 on average\(^{12}\). This indicates that our assumption of Weibull distributed lifetimes leads to lower marginal cost on average than would have been the case if we had assumed a deterministic lifetime function.

In the computation we use observed values for road width and HGV and passenger car traffic. We also utilize fitted pavement lifetimes. Together with the unit cost (65 SEK/sqm) the average costs during the pavement lifetime is 0.26 SEK/HGV km and 0.02 SEK per passenger car km. Observed costs data from VERA (SRA’s business system) indicates AC=0.47 SEK/HGV

\(^{10}\)Since available engineering assessments of the unit cost do not cover all regions, the average is not necessarily representative in all cases.

\(^{11}\)This is the official discount rate used by Swedish transport authorities (SIKA, 2002).

\(^{12}\)The 5:th and 95:th percentiles are 0.8230 and 0.8459, so this factor comes from a very narrow distribution.
Table 4: Estimated average and marginal costs (SEK/km)

<table>
<thead>
<tr>
<th></th>
<th>HGV:s</th>
<th>Pass. cars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\hat{AC})</td>
<td>(\hat{MC})</td>
</tr>
<tr>
<td>Mean</td>
<td>0.2555</td>
<td>0.0086</td>
</tr>
<tr>
<td>5:th percentile</td>
<td>0.0169</td>
<td>0.0006</td>
</tr>
<tr>
<td>10:th percentile</td>
<td>0.0230</td>
<td>0.0008</td>
</tr>
<tr>
<td>25:th percentile</td>
<td>0.0447</td>
<td>0.0015</td>
</tr>
<tr>
<td>50:th percentile</td>
<td>0.1135</td>
<td>0.0038</td>
</tr>
<tr>
<td>75:th percentile</td>
<td>0.2790</td>
<td>0.0094</td>
</tr>
<tr>
<td>90:th percentile</td>
<td>0.5852</td>
<td>0.0197</td>
</tr>
<tr>
<td>95:th percentile</td>
<td>0.8620</td>
<td>0.0291</td>
</tr>
</tbody>
</table>

km and \(AC=0.05\) SEK per passenger car km for paved road maintenance, but this cost category includes costs additional to pavement work. The unit costs used here are for pavement renewal only, so our AC estimates should be realistic. In addition it should be remembered that we only use one unit cost although this is known to vary. We proceed by computing the marginal costs, one value per observation in our data. Mean values and percentiles of estimated average and marginal costs are found in table 4.

The mean marginal cost estimate of HGV:s is 0.009 SEK/km, a very low value compared to the average cost. The computed percentiles reveal some dispersion but marginal costs are always low. The explanation for this is of course the low deterioration elasticity. For passenger cars the marginal cost is even lower, 0.0008 SEK/km. If one introduces a traffic dependent unit pavement cost the MC would probably be a higher share of AC than implied by the deterioration elasticity.\(^{13}\)

### 6 Conclusions

This paper develops a model for the relation between repavement costs and traffic flow. Based on an earlier model it adds greater realism. Using a large data set, it also estimates some model parameters and derives the marginal present value cost for road pavement renewal. The estimations are made with good results, the overall model fit is excellent and the coefficients are mostly statistically significant. The deterioration elasticities have the expected sign

\(^{13}\)Then one would introduce \(C = C(Q)\).
but are quite low (in absolute value terms). From the estimations it seems quite clear that the fundamental theorem does not apply to the Swedish national road network, since the deterioration elasticity for HGV:s is clearly different from -1. Higher traffic intensity is only to a minor part mirrored in shorter pavement lifetimes. As a consequence, marginal costs for HGV:s are found to be quite low and certainly lower than average costs. Earlier deviations from the fundamental theorem has been explained by deterioration caused by a time/weather effect. Here though, we are able to reject the hypothesis of time/weather having any influence on the pavement lifetime. As a result, the weak relation between traffic and pavement lifetime is even more remarkable. Given our theoretical framework, it is very hard to understand why road quality, which is assumed (and stated in official documents) to be decisive for repavement, should be that irresponsive to traffic volume. But one possible explanation is that the pavement standard of roads with high traffic intensity are higher, which should motivate a differentiation of the pavement unit cost. Another finding is that the impact from passenger cars on pavement lifetimes is significant, a result that contradicts the usual assumption that deterioration is caused by HGV:s (standard axes) only. Thus we get a MC for passenger cars as well, which is also lower than AC. Finally we find that using the model developed here, where pavement lifetimes are assumed to be Weibull distributed rather than deterministic, results in lower marginal costs.

We have made the present pavement lifetime, \( T \), stochastic and thus added some realism to the model. A remaining problem is, however, that future intervals are treated like fixed values. To remove this inconsistence would be a valuable improvement to the model and is a suggestion for future research. We also suggest a differentiation of the pavement unit cost, \( C \).

References


Ekdahl, P. (2000). *Deterioration models and road capital as tools in perfor-*
mance contracts for pavement maintenance. Doctoral thesis, Department of Technology and Society, Lund University.


Appendix

Let the function \( Q(t) \) be subject to a small disturbance \( \varepsilon \cdot L(t) \) in the interval \( t \in [t_1, t_2] \) where \( L(t) \) is zero outside the interval and where \( 0 < t_1 < t_2 < \min(T(Q), T(\overline{Q}(\varepsilon))) \). Let \( \overline{Q}(\varepsilon) \) be the average of the function \( Q(t) + \varepsilon \cdot L(t) \):

\[
\overline{Q}(\varepsilon) = \frac{1}{T(\overline{Q}(\varepsilon))} \int_0^{T(\overline{Q}(\varepsilon))} Q(t) + \varepsilon \cdot L(t) dt
\]

Thus:

\[
T(\overline{Q}(\varepsilon)) \cdot \overline{Q}(\varepsilon) = \int_0^{T(\overline{Q}(\varepsilon))} Q(t) dt + \varepsilon \cdot \alpha
\]

where:

\[
\alpha = \int_{t_1}^{t_2} L(t) dt
\]

Differentiation of both sides with respect to \( \varepsilon \) results in:

\[
T'(\overline{Q}(\varepsilon)) \cdot \frac{d\overline{Q}}{d\varepsilon} \cdot \overline{Q}(\varepsilon) + T(\overline{Q}(\varepsilon)) \cdot \frac{d\overline{Q}}{d\varepsilon} = Q(T(\overline{Q}(\varepsilon))) \cdot T'(\overline{Q}(\varepsilon)) \cdot \frac{d\overline{Q}}{d\varepsilon} + \alpha
\]

Letting \( \varepsilon \to 0 \) we get the general formula:

\[
\frac{d\overline{Q}}{d\varepsilon} = \frac{\alpha}{T(\overline{Q}) + T'(\overline{Q}) \cdot \overline{Q} - T'(\overline{Q}) \cdot Q(T(\overline{Q}))}
\]

Let \( t_2 - t_1 = 1 \) and \( L(t) = 1 \). Then \( \frac{d\overline{Q}}{d\varepsilon} = \frac{1}{T(\overline{Q})} \) when \( Q(T) \) equals \( \overline{Q} \) (or when \( T'(\overline{Q}) = 0 \)).