The welfare-maximizing discount rate in a small open economy

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Abstract

The controversy about what approach is best for calculating the social discount rate for public investments is both long standing and heated. Two main approaches are the social time preference and the social opportunity cost approaches. Complicating issues are tax wedges, the question of whether public investments crowd out current private consumption or private saving, and the possibility of myopic behavior among individuals. This study uses a model that takes these issues into account to derive the discount rate that optimizes welfare in a small open economy. The result is that even if individuals have behavioral preferences differing from the normative preferences of the social planner and even if tax wedges exist, the optimal discount rate is the pre-tax market return on capital, as long as individuals are forward looking and consistent in their behavioral preferences, and their choices are not constrained by, for example, liquidity restrictions.

Keywords

Discounting; Tax wedge; Social time preference; Social opportunity cost

JEL Codes

E21; F41; H43

Svensk sammanfattning

Kontroversen om vilket tillvägagångssätt som är mest lämpat för beräkning av diskonteringsräntan för offentliga investeringar (t.ex. infrastrukturinvesteringar) är både långlivad och levande. Två huvudsakliga tillvägagångssätt är den sociala tidspreferensansatsen och alternativkostnadsansatsen. Komplicerande faktorer är skattekilar, frågan om offentliga investeringar tränger ut nuvarande privat konsumtion eller privat sparande och möjligheten att individerna beter sig kortsiktigt. I denna studie används en modell som tar hänsyn till dessa frågor för att härleda den diskonteringsränta som maximerar välfärden i en liten öppen ekonomi. Resultatet är att även om individer har andra beteendepreferenser jämfört med riksdagens normativa preferenser och om skattekilar existerar, är den optimala diskonteringsräntan lika med marknadsavkastningen på kapital före skatt.

I denna analys har det antagits att individer är rationella i den meningen att de är framåtblickande och konsekventa i preferenser över de analyserade tidsperioderna, och att deras val inte är begränsade av till exempel likviditetsrestriktioner. Som ett resultat förutspår modellen att om en ytterligare offentlig investering finansieras av inkomstskatter (t.ex. på arbetsinkomst), kommer effekten på landets samlade sparande att kompenseras av en minskning av det privata sparandet, dvs. staten kan inte öka det totala sparandet genom att öka mängden offentliga investeringar. Den välfärdsmaximerande diskonteringsräntan för offentliga investeringar är därför lika med avkastningen på privata investeringar.

Men om individerna i själva verket inte är rationella (eller om de är begränsade av likviditetsrestriktioner) och inte justerar sina sparbeslut när offentliga investeringar ökar, vad skulle det innebära? Om riksdag och regering då skulle vilja öka medborgarnas välbefinnande genom att öka offentliga investeringar, skulle det fortfarande vara optimalt att välja de investeringar som ger högst avkastning. Varför skulle staten under sådana omständigheter inte välja att göra direkta investeringar på finansmarknaden om den kan få en högre avkastning där, jämfört med från traditionella offentliga investeringar såsom infrastrukturinvesteringar? Om man accepterar att sådana finansiella offentliga investeringar är ett alternativ, är återigen alternativkostnaden som motsvarar diskonteringsräntan lika med den marknadsmässiga avkastningen på kapital före skatt.

Den genomsnittliga reala avkastningen för finansiella investeringar i Sverige med motsvarande risknivå som transportinfrastruktur i Sverige har också estimerats i en mindre kompletterande analys för åren 2018–2021. Detta ger ett intervall på 6,6% - 7,5% årligen, med ett genomsnitt på 6,9%, vilket är det bästa estimatet på den reala diskonteringsräntan för transportinfrastrukturinvesteringar i Sverige år 2022 från denna studie.

1 Introduction

The social discount rate (SDR) could generally be viewed as the shadow price of a one-year delay in realizing the social costs and benefits in the cost–benefit analysis of public investments. It ultimately determines how many resources should optimally be spent on public investments, and how to rank projects with various time horizons. Which approach is more suitable for calculating the SDR is both a long-standing and heated controversy. Two main approaches are the social time preference (STP) approach and the social opportunity cost (SOC) approach. Complicating issues are tax wedges, the question of whether public investments crowd out current private consumption or private saving, and the possibility of myopic behavior among individuals. This study uses a model that takes these issues into account to derive the discount rate that optimizes welfare in a small open economy. The approach essentially entails a microeconomic model of the tradeoffs between public investments and the savings decisions of individuals.

The problem of finding the appropriate discount rate has long puzzled welfare economists and resulted in a protracted controversy well covered in a vast literature. Consequently, empirical estimates have had large ranges (e.g., Baumol, 1968, noted that various government agencies used ranges of 0–8%), resulting in vastly different investment advice. Much of the theoretical literature in the field is based on American conditions, and discussion and practice in the USA are still based on Harberer's analysis of a closed economy. As shown by Sandmo and Drèze (1971), however, the conditions are essentially different in a small open economy. Although there is a comprehensive discounting literature, relevant analyses have been scarce. Also, Sandmo and Drèze (1971) used the closed economy as the baseline analysis, although this assumption was later relaxed. The present study derives the welfare-maximizing discount rate for public investment in a small open economy. The focus on a small open economy allows us to arrive at a closed-form solution, more easily estimated in practice than the resulting formulas of Sandmo and Drèze (1971), which relied on elasticities that are hard to estimate in a controlled manner. A difference between the closed economy case and the small open economy case is that a production focus is more relevant in the former and a consumer focus is more relevant in the latter (as the capital stock available for firms is not determined by domestic saving). Consequently, Sandmo and Drèze (1971) had more of a production focus, while the present study has more of a consumer focus, somewhat generalizing Sandmo and Drèze's (1971) model in such dimensions (e.g., distinguishing between the consumption discount rate and the utility discount rate, as is standard in the STP tradition).

The model in this study is largely similar to that of Liu (2003), who introduced a marginal cost of funds (MCF) approach to multi-period project evaluation and derived a decision criterion. However, there are some key differences. Most importantly, while Liu included an endogenous labor supply, this study (and that of Sandmo and Dréze, 1971) uses the less realistic assumption of an exogenous labor supply. However, in doing so, a simple closed-form solution is achieved here, while this was not accomplished by either Liu (2003) or Sandmo and Dréze (1971), whose resulting formulas were rather complicated and, more importantly, hard to estimate in practice. In fact, Liu relied on the assumption that the indirect effect of each project on the government budget could be estimated, which is not straightforward in practice. Also, the MCF as such (disregarding the time aspect) is a complicated and disputed concept. For example, Jacobs (2008) showed that when distributional considerations are taken into account, in line with the modern tradition of optimal taxation (starting with Mirrlees, 1971), the MCF equals unity. Regarding the example of Liu's work, Liu (2003) and Burgess (2013) disagreed on the interpretation of the MCF, with Liu claiming that his metric was in line with the usual interpretation, while Burgess used formal analysis to argue that this was not the case. Moreover, Burgess (2013) demonstrated that an analysis of the welfare-maximizing discount rate in a setting with an exogenous labor supply could be complemented by the MCF to account for deadweight losses of labor taxation in a separate step, and that this would yield the same policy advice as would Liu's integrated approach. The present analysis is therefore limited to the more straightforward task of finding the welfare-maximizing discount rate given an exogenous labor supply, leaving questions about the MCF to other studies.

Another difference is that in this analysis a more general welfare function is chosen than that chosen by Liu (2003) or Sandmo and Dréze (1971), not assuming that the relative valuation of consumption in different periods coincides with that of the individuals (in line with the findings of Laibson, 1997). My assessment is that this project is unique in distinguishing between the descriptive preferences expressed in individual behavior, on one hand, and the normative preferences of the social planner, on the other, while using a nested optimization structure as is standard in the optimal taxation literature (e.g., Mirrlees, 1971) but often absent from the discounting literature.

The aim is to find the SDR for investments in a public good, for example, open-access infrastructure investments, when such investments are financed by a tax on current income (i.e., labor income in period 1), in a small open economy. That is, this analysis focuses on the special case of a setting in which public investments are financed by a tax on current consumption, corresponding, for example, to how infrastructure investments have traditionally been financed in Sweden. This special case also contrasts with the analysis of Sandmo and Drèze (1971), who did not include labor taxes but analyzed the special case of financing public investments by increased capital taxes.

The paper is organized as follows. In the next section, extended background is given, discussing the STP and SOC methods, justifying the welfare function, and previewing the results. In section 3, the relevant theoretical literature is reviewed, and in section 4, the theoretical analysis is developed. In section 5, the conclusions from section 4 are applied to the case of Swedish transport infrastructure investments. Finally, section 6 discusses implications and concludes the paper.

2 Background

There are two basic rationales for the existence of the SDR, i.e., the opportunity cost argument and the utility argument, implying two methods for estimating the discount rate: the SOC method and the rate of STP method, respectively. The opportunity cost argument views public investments as displacing either private consumption or private investment, which constitutes an opportunity cost. Another possible interpretation of SOC is that if there are other public investment opportunities that will produce expected positive returns, then such investments should be chosen instead of investments not expected to produce such returns. The utility-based argument rests on the assumption of a cardinal (concave) utility function, i.e., as people get richer, their marginal utility of consumption decreases. As we today expect per capita consumption to grow, it is rational to put less value on future than present consumption. The dilemma of choosing between these two approaches was summarized by Bradford (1975):

> On the one hand it would seem there is a clear opportunity for welfare gain in undertaking an investment with rate of return in excess of a social time preference rate, however determined. On the other hand, it would clearly be possible to do even better by leaving the resources in the private sector if the rate of return on private investment exceeded the return on the government project. (Bradford, 1975, p. 887)

How to choose between the two approaches remains an unresolved controversy (see, e.g., Burgess and Zerbe, 2011; Moore et al., 2013a,b; Spackman, 2020). A key issue in general is how to account for distortionary taxes in a proper way. Both the STP and SOC methods must account for the degree to which public investment displaces private investment. In the STP approach, this can be done using the shadow cost of capital approach. In this approach, the STP rate is the individual time preference, which is equal to after-tax market returns, while the SOC rate is the pretax market return. Alternatively, a starting point for such an adjustment in an SOC setting is the following simple formula:

$$\text{SDR} = \alpha\text{ROI} + \mu\text{CRI} + \gamma\text{FB}, \qquad (1)$$

where α, μ, and γ denote the proportions of funds from displaced private-sector investment, forgone consumption, and foreign borrowing, respectively, ROI is the marginal before-tax return on displaced private-sector investment, CRI is the after-tax return on saving, and FB is the real marginal cost of incremental foreign borrowing (see, e.g., Moore et al., 2013a). Sjaastad and Wisecarver (1977) showed that, under basic conditions, the shadow price of capital adjustment to the STP approach will yield the same answer as does the SOC approach.

Liu (2003) argued that both the weighted average approach (eq. [1]) and the shadow price of capital approach suffered from severe implementation problems, since no general formula for how to estimate the fractions drawn from consumption versus private investment exists and since the SDRs in these approaches are project specific. However, Liu did not solve this problem, since his approach assumed that the indirect budget effects of each project could be estimated (but did not specify how). Moore et al. (2013b) noted that it is tricky to determine to what extent public investment crowds out private consumption versus private investment, and according to OECD (2018, p. 221), such adjustments to the STP method is seldom implemented in practice due to extensive informational requirements and a lack of generally accepted approximations. However, in this study, the resulting formula of the SDR is both rather simple to estimate in practice (as demonstrated in section 5) and is project independent.

It has also been claimed that when public investments are financed by taxes, the simple STP approach is sufficient. For example, Moore et al. (2013a, pp. 10–11) argued that taxes primarily reduce current consumption rather than private investment for the simple reason that consumption is much larger than investment (typically five times as large) and, therefore, taxes on investment cannot yield as much as taxes on consumption.

A further implication of the assumption that government projects are funded mainly by taxes is that, if one uses the STP method, there is little need to shadow price private-sector investment. Under these circumstances discounting becomes easy: analysts should simply discount using the rate of STP.

In the present analysis, I will show that these statements are incorrect, intuitive and logical as they may seem. What is the STP approach really about? Let us now turn specifically to that question. The STP approach for discounting rests on the assumption of a cardinal utility function.[1] Assuming that the utility of individuals is a continuous function of their consumption, their marginal utility from additional consumption can be estimated by differentiating that same utility function.

The elasticity of marginal utility of consumption (EMUC):

$$\eta = -\frac{c \cdot \frac{\partial^2 u}{\partial c^2}}{\frac{\partial u}{\partial c}}, \tag{2}$$

has in numerous empirical studies been found to be positive, implying a diminishing marginal utility from increased consumption. Hence, the so-called Ramsey rule (attributed to Ramsey, 1928) of discounting is:

$$r = \delta + \eta \cdot g, \tag{3}$$

where r is the (real) discount rate (social or individual), δ is the pure rate of time preference (PRTP), and g is the projected growth rate of consumption (typically based on the growth rate of production). Historically, the global economy has experienced a positive average growth rate for a long time (i.e., centuries), so it is reasonable to expect continued growth. Therefore, the Ramsey rule is based on the idea that, on average, each individual is expected to be richer tomorrow than today, and accordingly one extra dollar tomorrow will give less utility than one dollar received today.

Baum (2009) reasoned that eq. (3) can represent both a normative formula for society and a descriptive model of individual behavior, but with distinct parameter values. If individuals are self-interested and rational, they will have a pure rate of time preference based on their perceived yearly risk of dying. However, if society also cares for future generations' wellbeing, it is not rational to base the social values of δ on the mean individual values. Instead, Dasgupta and Heal (1979) argued that they should be based on the analogous social hazard rate, i.e., the risk of societal extinction. A related factor that may explain differences

[1] Cardinal utility has long been considered outdated in most other branches of economic theory, since the idea imposes somewhat restrictive assumptions that are not always necessary for economic analysis. However, as early as 1936, Alt showed that relatively simple assumptions (accepted and used by some ordinalists) were sufficient to imply a cardinal utility function, and this concept has recently attracted some revived interest among theorists (see Köbberling, 2006).

between SOC and STP is the possibility of under-saving by individuals due to limitations in self-control, suggested by some literature.[2] Boadway (2006) reasoned that this raised unsolved questions about adopting paternalism.

The approach chosen here is general enough to allow the parameter values for the social planner to differ from the individuals' values (with or without paternalistic motives). However, even though the parameters in eq. (3) are part of both the individuals' and the government's respective optimization problems, it is shown that they are irrelevant to determining the SDR in a small open economy when public investments are financed by tax on current income. Instead, the result is that even if individuals have behavioral preferences differing from the normative preferences of the social planner and even if tax wedges exist, the welfare-maximizing discount rate is the pre-tax market return on capital, as long as the individuals are forward looking and consistent in their behavioral preferences, and their choices are not constrained by, for example, liquidity restrictions. A further strength of the result is that a welfare-optimal baseline policy is not required for it to hold.

3 Literature

The literature on discounting is vast. This section employs a narrow focus on discounting, concentrating solely on the literature that is most relevant to the subsequent analysis, i.e., a welfare theoretic focus on public investments, taking alternative costs of financing into account but ignoring distributional aspects and systematic risk.[3] For a broader overview of discounting in theory and practice, see OECD (2018) and Spackman (2020).

An early contribution with canonical influence is "A mathematical theory of saving" by Frank Ramsey (1928). Using a welfare theoretic approach, he examined how much a nation should save, and came up with the following rule of thumb:

> The rate of saving multiplied by the marginal utility of money should always be equal to the amount by which the total net rate of enjoyment of utility falls short of the maximum possible rate of enjoyment. (Ramsey, 1928, p. 543)

This was later translated into the Ramsey rule of discounting (eq. [3]), constituting the foundation of the STP approach, but later work has greatly complicated our understanding of the issue. Marglin (1963) and others have argued that current consumers may not be representative of how society values the future, i.e., there may be two discount rates, the private and the social discount rates, that need not coincide. Baumol (1968) argued that there are two private discount rates, of firms and individuals, and that this wedge is caused by corporate taxes. Sandmo and Drèze (1971) elaborated on this, using the simple case of a closed economy. They examined what effect public investments financed by capital taxes would have on private saving and thereby firm output, deriving the welfare-maximizing discount rate from this. They also extended the model with foreign borrowing, to also study the open economy case. However, they did not include other types of taxes than capital taxes,

[2] Laibson's seminal analysis from 1997 proposed that lack of self-control could influence the saving decisions of individuals in an adverse way. In neuroscience it has been shown that various types of addictions or medial orbitofrontal damage are associated with myopic discounting (Sellitto et al., 2010). Howard (2013) found that individuals are considerably more impatient with personal maturities than with charity maturities.

[3] Systematic risk is briefly described in section 4.

and their final formulas were not in a closed form but included various elasticities that are hard to estimate in a controlled manner. An even more ambitious model was formulated by Pestieau (1974), in which taxes on both interest income and labor income were included as well as debt in conjunction with public investment (for a closed economy). However, due to the high complexity, he made less progress in solving the problem, but concluded from the first-order conditions that an optimal policy of taxation and public capital accumulation was apparently that which sets the tax rates according to Ramsey's optimal taxation structure and that equates the rate of return on public investment to the STP rate.

Liu (2003), however, partly solved the discount problem with an endogenous labor supply, using what he called a "marginal cost of funds" (MCF) approach. However, the resulting formula was rather complex. In Liu's own words (p. 1715):

> According to criterion (15), the MCF approach to multi-period project evaluation consists of the following components. (i) A project should be represented as a stream of direct investments, a stream of direct benefits measured as contemporaneous willingness to pay and a stream of indirect revenue benefits; (ii) future project direct benefits should be discounted at the net rate of return while future project costs, including indirect revenue benefits as negative costs, should be discounted at the gross rate of return; (iii) the present value of net costs should be multiplied by the MCF before being compared to the present value of the direct benefits.

That is, the problem of finding the indirect revenue benefits remained, and not just "once and for all" when calculating the MCF, but for each year of every specific project under consideration, which seems impractical and calls for some additional theory. Also, there has been disagreement about interpreting the MCF factor from Liu's model. Liu (2003) claimed that it was interpreted as usual, while Burgess (2013) disagreed, using formal modeling as a basis for his stance. Burgess discussed and further developed Liu's model to show that, with some simple adjustments, the traditional SOC approach was consistent with Liu's MCF approach.

Otherwise, the theoretical focus has recently been more on other aspects of discounting such as systematic risk (see section 5 and Gollier, 2014) and distributional aspects (which is more relevant to climate change valuation; see, e.g., Fleurbaey and Zuber, 2015). However, discussion of whether the SOC or STP approach is more appropriate is still lively (see, e.g., Burgess and Zerbe, 2011; Moore et al., 2013a,b; and Spackman, 2020).

4 Theory

The following analysis seeks to find the SDR for investments in public goods, for example, open-access infrastructure investments, in a small open economy. A key reference for such an analysis is the work of Sandmo and Drèze (1971), who analyzed this problem in a closed economy case, with a subsequent analysis of an open economy case. Following Sandmo and Drèze (1971), the present analysis uses a nested optimization structure, distinguishing between the private market equilibrium and the welfare optimum (defined by the social planner). That is, the social planner (i.e., government) takes the private equilibrium (which depends on public investment decisions) as given when maximizing welfare, according to its own sovereign definition.

A further difference between the closed economy case and the small open economy case is that a production focus is more relevant in the former and a consumer focus is more relevant in the latter, as the capital stock available for firms is not determined by domestic saving. The private return on capital will be determined by capital supply and demand on the international market and can therefore be treated as exogenous (see, e.g., Mankiw, 2000). Consequently, Sandmo and Drèze (1971) had more of a production focus, modeling firm production, while this dimension is absent from the present analysis, in which a pure individual choice model is used to represent the relevant private market. Instead, the model is somewhat generalized (compared with that of Sandmo and Drèze, 1971) in the dimensions determining such choices (e.g., distinguishing between the consumption and utility discount rates, as is standard in the STP tradition). Distortionary taxes on first- and second-period incomes (e.g., labor incomes) as well as on capital returns are included. However, a simplification (as used by Sandmo and Drèze, 1971) that considerably reduces the amount of algebra is that the labor supply is treated as exogenous. Sandmo and Drèze (1971) did not include labor taxation in their model, and consequently assumed that public investments were financed by capital taxes. Here I study another special case, in which the capital tax rate is held fixed but the capital tax income (accruing to the government) varies with private saving rates. Hence, public investments are financed by tax increases on income in period 1, while a reduction in capital tax incomes is financed by tax increases on income in period 2.[4] A key assumption regarding the individual choice equilibrium is that an interior solution is reached, i.e., the representative individual is not constrained in her choice by, for example, liquidity restrictions.

Next the model is formally defined (sections 3.1 and 3.2) and the SDR is derived (sections 3.3 and 3.4). For more detailed derivations, see Appendix A.

4.1 The individual's problem

A representative individual faces the following utility maximization problem during two periods (subscripts 1 and 2):

$$\max_{c_t, S_p} U(c_1, c_2) = u(c_1) + u(c_2) \cdot \beta_p \tag{4}$$

s.t.

$$c_1 = \left(1 - \tau_{y,1}\right) \cdot y_1 - S_p + \varepsilon_{p,1}, \tag{5}$$

$$c_2 = \left(1 - \tau_{y,2}\right) \cdot y_2 + R_p \cdot S_p + R_s \cdot S_s + \varepsilon_{p,2}. \tag{6}$$

[4] I.e., no additional borrowing from the public is allowed to finance public investments, but such an analysis would constitute an additional special case.

$R_p = \left(1 + (1 - \tau_K) \cdot r_p\right)$ is the after-tax factor return on private investments.

$R_s = (1 + r_s)$ is the factor return on public investments.

c_t is consumption in period t, with: $0 < c_t < \infty$.

β_p is the private utility discount factor (corresponding to the private PRTP), with:

$0 < \beta_p \leq 1$.

y_t is taxable non-capital income (e.g., labor income) in period t, here assumed to be exogenous, with:

$0 < y_t < \infty, y_1 \leq y_2$.

r_p denotes the exogenous rate of return on private investments, with:

$0 \leq r_p < \infty$.

r_s denotes the exogenous rate of return on public (social) investments, with:

$0 \leq r_s < \infty$.

S_p is the endogenous (private) saving between the two periods, with:

$-\infty < S_p < \infty$.

S_S is the public (social) investment (saving) per capita between the two periods, which is exogenous from the individual's point of view, with:

$0 \leq S_s \leq \widehat{S_s}$, where $\widehat{S_s}$ is an exogenous positive number, i.e., corresponding to the maximum available number of potential projects with return r_s.

$\varepsilon_{p,t}$ is the residual in period t, i.e., the net disposal in the analyzed periods from ingoing capital (in period 1) and (non-taxable) government transfers and outgoing capital (in period 2, i.e., it includes savings in period 2 for future periods not formally modeled). $\varepsilon_{p,t}$ is treated as exogenous, with:

$-\infty < \varepsilon_{P,t} < \infty$.

$\tau_{y,t}$ denotes the tax rate on taxable non-capital income (e.g., labor income) in period t, with: $0 \leq \tau_{y,t} < 1$.

τ_K denotes the tax rate on capital (e.g., labor income) in period t, with: $0 \leq \tau_K < 1$.

A difference from the conventional framing in welfare economics is that $U(c_1, c_2)$ represents the behavioral preferences of the representative agent, i.e., the implicit utility function that can be derived from how she acts, but need not represent the utility function that ultimately makes her truly happy or satisfied, i.e., individuals are not restricted to being completely rational according to the conventional definition, which represents a generalization. However, the representative agent is still assumed to be forward looking and consistent in her preferences regarding states of the world (in each period analyzed), so that the behavioral utility function is independent of, for example, government policy – i.e., there is still some basic rationality in the behavioral utility function. However, the two-period model means that preferences are not restricted to staying constant over long time horizons, but only over the analyzed periods, i.e., this is also a generalization compared to infinite time models. To conclude, the model of individual behavior represents a purely descriptive, not normative, model; the normative utility function is instead represented by the social planner's problem.

A key assumption is that $u(c_t)$ is a differentiable function and that $u'(c_t)$ is a homogeneous function, but otherwise few assumptions as to $u(c_t)$ are needed. For simplicity, let us assume the familiar isoelastic utility function (with constant relative risk aversion):

$$u(c_t) = \frac{{c_t}^{1-\eta_p} - 1}{1-\eta_p} \qquad (7)$$

$$\eta_p \neq 1$$

where η_p is the private (behavioral) elasticity of marginal utility of consumption (EMUC). Normally, EMUC is assumed to be positive, but this assumption is not necessary for the following algebraic derivations and results. Please note that if $\eta_p = 1$, then $u(c_t) = \ln(c_t)$.

It follows from eq. (7) that:

$$u'(c_t) = {c_t}^{-\eta_p} \; . \qquad (8)$$

Let us now turn to the social planner's problem.

4.2 The social planner's problem

The social planner, i.e., the government, faces the following welfare maximization problem:

$$\max_{S_s, \tau_{y,1}, \tau_{y,2}} W(c_1, c_2) = v(c_1) + v(c_2) \cdot \beta_s \qquad (9)$$

s.t.

$$\tau_{y,1} \cdot y_1 = S_s + \varepsilon_{s,1}, \qquad (10)$$

$$\tau_{y,2} \cdot y_2 + \tau_K \cdot r_p \cdot S_p = \varepsilon_{s,2}. \qquad (11)$$

c_1, c_2, S_p are functions of $S_s, \tau_{y,1}, \tau_{y,2}$ (according to the individual equilibrium).

τ_K, r_s are exogenous.

$\varepsilon_{S,t}$ is the (exogenous) government expenditure per capita in each period t (net of revenues other than from taxation), with:

$-\infty < \varepsilon_{s,t} < \infty$.

β_s is the social utility discount factor, corresponding to the social PRTP, with

$0 < \beta_s \leq 1$.

Equations (10) and (11) are the public budget restrictions in each period and define how public investments are financed. As defined here, no extra public lending is allowed to finance public investments, but these equations imply that the reforms should be budget neutral in each period, i.e., a balanced budget restriction applies in each period. For the first period, this implies that public investments are financed with an increased tax on income (e.g., on labor income) in the first period. For the second period, the implication is that if more public investment leads to lower individual savings, this will lead to lower tax revenue from the capital tax, which will need to be compensated for by an increased tax on income (e.g., labor income) in the second period.

When it comes to the utility function, again, a key assumption is that $v(c_t)$ is a differentiable function and that $v'(c_t)$ is a homogeneous function (see eq. [25]), but otherwise few assumptions as to $v(c_t)$ are needed. Again, for simplicity let:

$$v(c_t) = \frac{c_t^{1-\eta_s} - 1}{1-\eta_s} \quad (12)$$

where η_s is the social EMUC.

$$v'(c_t) = c_t^{-\eta_s} \quad (13)$$

The problem defined by eqs. (9)–(11) can be rearranged as follows:

c_1, c_2 are dependent variables of $\tau_{y,1}$, $\tau_{y,2}$, S_s,

$\tau_{y,1}$, $\tau_{y,2}$ are dependent variables of S_s, and

S_s is the only independent variable.

Let us now turn to the equilibria implied by this model.

4.3 The individual's equilibrium

Maximization of eq. (1) gives:

$$u'(c_1) = u'(c_2) \cdot \beta_p \cdot \left(1 + (1 - \tau_K) \cdot r_p\right) \quad (14)$$

$$c_1 = \left(1 - \tau_{y,1}\right) \cdot y_1 - S_p + \varepsilon_{p,1}, \quad (15)$$

$$c_2 = \left(1 - \tau_{y,2}\right) \cdot y_2 + R_p \cdot S_p + (1 + r_s) \cdot S_s + \varepsilon_{p,2}, \quad (16)$$

where eq. (14) is the (after-tax) Euler equation (which can be rearranged to give the marginal rate of substitution[5]). From equations (14)–(16), c_1, c_2, S_p can be solved as functions of only $S_s, \tau_{y,1}, \tau_{y,2}$.

Inserting eq. (8) into eq. (14) gives:[6]

$$c_2 = c_1 \cdot G, \quad (17)$$

where G is the growth factor of consumption:

$$G = \left[\beta_p \cdot R_p\right]^{\frac{1}{\eta_p}}, \quad (18)$$

where $\frac{1}{\eta_p}$ is the elasticity of intertemporal substitution (EIS).

G is exogenous (because all ingoing parameters are exogenous), so the proportion between c_1 and c_2 is independent of $S_s, \tau_{y,1}$, and $\tau_{y,2}$. Hence, the government cannot affect the total savings rate (i.e., based on the sum of private and public savings) by adjusting the public saving. This result is parallel to the concept of Ricardian equivalence, i.e., that the government cannot change the proportion between c_1 and c_2 by altering that between $\tau_{y,1}$ and $\tau_{y,2}$ in a model with government borrowing and lending.

Proposition 1: If the government does not adjust the capital tax when the amount of public investment changes, then the total savings rate (i.e., based on the sum of private and public savings) is fixed and independent of such changes in public investments.

From this one may suspect that the SOC of the SDR, r_s^*, equals the return on private investment, r_p. In the following, it will be formally shown that $r_s^* = r_p$ maximizes welfare, irrespective of social and private preferences, and irrespective of whether or not the current level of public savings rate is optimal.

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[5] $u'(c_1)/\left(u'(c_2) \cdot \beta_p\right) = 1 + (1 - \tau_K) \cdot r_p$

[6] It is easy to see that a key assumption needed for eq. (14) to take this form is that $u'(c_t)$ is a homogeneous function.

Using eq. (17), c_1, c_2, S_p can be solved as functions of only $S_s, \tau_{y,1}, \tau_{y,2}$ (see Appendix A).

$$S_p(S_s, \tau_{y,1}, \tau_{y,2}) = \frac{-[(1-\tau_{y,2})\cdot y_2 + R_s \cdot S_s + \varepsilon_{p,2}] + [(1-\tau_{y,1})\cdot y_1 + \varepsilon_{p,1}]\cdot G}{G + R_p} \quad (19)$$

$$c_1(S_s, \tau_{y,1}, \tau_{y,2}) =$$
$$(1-\tau_{y,1})\cdot w_1 L_1 + \frac{[(1-\tau_{y,2})\cdot y_2 + R_s \cdot S_s + \varepsilon_{p,2}] - [(1-\tau_{y,1})\cdot y_1 + \varepsilon_{p,1}]\cdot G}{G + R_p} + \varepsilon_{p,1} \quad (20)$$

$$c_2(S_s, \tau_{y,1}, \tau_{y,2}) = (1-\tau_{y,2})\cdot y_2 +$$
$$+ \frac{-[(1-\tau_{y,2})\cdot y_2 + R_s \cdot S_s + \varepsilon_{p,2}] + [(1-\tau_{y,1})\cdot y_1 + \varepsilon_{p,1}]\cdot G}{\frac{G}{R_p} + 1} + R_s \cdot S_s + \varepsilon_{p,2} \quad (21)$$

4.4 The welfare-maximizing discount rate

To find the SDR r_s^* that leads to welfare-maximizing choices when making public investment decisions, we need not solve the optimization problem defined in eqs. (6)–(9). The welfare-maximizing SDR r_s^* is here defined as the lowest return on public investments, r_s, that makes an additional (marginal) public investment increase welfare. The lower limit of r_s is thus the value of r_s that makes an additional public investment have no effect on welfare:

$$r_s^* = r_s$$

where r_s solves:

$$\frac{dW}{dS_s} = 0, \quad (22)$$

provided that the effects on the private equilibrium and the budget constraints (eqs. [10] and [11]) are taken into account.

$$\frac{dW}{dS_s} = v'(c_1)\cdot\frac{dc_1}{dS_s} + v'(c_2)\cdot\frac{dc_2}{dS_s}\cdot\beta_s \quad (23)$$

From the individual's equilibrium (eq. 17) we have:

$$c_2 = c_1 \cdot G\ .$$

Assuming that c_1, c_2 are continuous and differentiable functions of S_s (on the relevant interval), and that eq. (17) holds for all S_s, differentiation is possible:

$$\frac{dc_2}{dS_s} = \frac{dc_1}{dS_s} \cdot G \ . \tag{24}$$

We also have $v'(c_t) = c_t{}^{-\eta_s}$ from eq. (13).

Inserting eqs. (13), (17), and (24) into eq. (23)[7] gives the new equilibrium condition:

$$\frac{dc_1}{dS_s} \cdot (1 + G^{1-\eta_s} \cdot \beta_s) = 0 \tag{25}$$

As g and β_s are positive (according to the definitions of the ingoing parameters), this gives $(1 + G^{1-\eta_s} \cdot \beta_s) \neq 0$, so eq. (25) implies:

$$\frac{dc_1}{dS_s} = 0 \ , \tag{26}$$

i.e., we only need to find $\frac{dc_1}{dS_s}$. Differentiation and simplification of eq. (20) gives:

$$\frac{dc_1\left(S_s, \tau_{y,1}(S_s), \tau_{y,2}(S_s)\right)}{dS_s} = \frac{1}{G+R_p} \cdot \left[R_s - y_1 \cdot R_p \cdot \frac{d\tau_{y,1}}{dS_s} - y_2 \cdot \frac{d\tau_{y,2}}{dS_s}\right]. \tag{27}$$

From the constraints (i.e., eqs. [10] and [11]) we can derive expressions of:

$$\tau_{y,1}(S_s) = \frac{S_s + \varepsilon_{s,1}}{y_1} \tag{28}$$

$$\tau_{y,2}(S_s) = \frac{\varepsilon_{S,2} \cdot (G+R_p) + \tau_K \cdot r_p \cdot \{y_2 + R_s \cdot S_s + \varepsilon_{P,2} - [y_1 - S_s - \varepsilon_{S,1} + \varepsilon_{p,1}] \cdot G\}}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \ . \tag{29}$$

Differentiation of eqs. (28) and (29) gives:

---

[7] Again, it is easy to see that a key assumption needed for eq. (20) to take this form is that $v'(c_t)$ is a homogeneous function.

$$\frac{d\tau_{y,1}}{dS_s} = \frac{1}{y_1} \tag{30}$$

$$\frac{d\tau_{y,2}}{dS_s} = \frac{\tau_K \cdot r_p \cdot (R_s + G)}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \ . \tag{31}$$

Inserting eqs. (30) and (31) into eqs. (26) and (27) and solving for $r_s = R_s - 1$ gives the equilibrium condition:

$$\frac{dc_1}{dS_s} = \frac{r_s - r_p}{2 + g + r_p} = 0 \ . \tag{32}$$

As $1 + g = G = \frac{c_2}{c_1} > 0$ and $r_p \geq 0$, we can conclude that the denominator is strictly larger than 0 and hence:

$$r_s{}^* = r_p \ . \tag{33}$$

Proposition 2: With utility functions (of the individuals and the social planner, respectively), with first-order derivatives that are homogeneous in consumption in each period and consistent over the analyzed time interval, and with a capital tax whose rate is independent of public investments, the welfare-maximizing SDR equals the pre-tax return on private saving, irrespective of the parameter values, including those of the utility function and the social welfare function.

This result corresponds to the result in one of the limiting cases for one of the resulting equations (eq. 21) of Sandmo and Dreze (1971) for a closed economy: $r_{preference} < r_s{}^* < r_p$.

5 An empirical example

In this section the results from section 3 are applied to the case of Swedish transport infrastructure investments. The theory section suggests that the SDR for public investments in a small open economy equals the pre-tax market rate of return on capital. How should this return be estimated? Clearly there are many different market rates of return for various assets, depending crucially on how risky each asset is. The return on so-called risk-free assets, such as government bonds issued by economically stable states, is typically much lower than the return on risky assets such as company shares.

Arrow and Lind (1970) wrote a canonical paper demonstrating that, under certain assumptions, the social cost of risk moves to zero as the population tends to infinity, so that public projects can be evaluated based on expected net benefit alone, and hence the risk-free discount rate (e.g., the market return on risk-free assets) can appropriately be used. However, according to Baumstark and Gollier (2014), there is no reason to believe that one of the fundamental assumptions holds in reality, namely, that the expected payoffs of public projects are uncorrelated with general consumption. Even if this were true on an aggregate level, various parts of the public sector would have different risk profiles.[8] Not taking this into account would yield a second-best strategy.[9] Recently, there has been rapid theoretical progress concerning the incorporation of risk into long-term SDRs, spurred by climate change economics. Gollier (2014) succinctly summarized the concluding result of a series of papers, i.e., a generalized version of the famous capital asset pricing model (CAPM) discount formula (attributed to Lucas, 1978), applicable to long-term social discounting (see eq. [3] in Gollier, 2014). When consumption follows a geometric Brownian motion, i.e., each year's growth rate is i.i.d. normally distributed, this expression collapses into the classical CAPM structure, which is a natural starting point for incorporating risk into the estimation of discount rates. The CAPM comes in two versions, one original asset price version and one consumption-based version. The structure of the model is the same in the two versions, but the empirical baseline differs, which means that the parameter values will also differ.

CAPM implies that the expected risk premium on a risky asset (e.g., a company or partial portfolio), defined as the expected return on a risky asset less the risk-free return, is proportional to the covariance of its return and the basis of total systematic risk (i.e., a perfectly diversified asset portfolio in the original model, consumption in the consumption-based version):

$$r = r_f + \beta \cdot \pi \,, \qquad (34)$$

where

r = expected return on security or portfolio,

r_f = risk-free interest rate,

β = correlation between the payoff of the asset in question and total basis (e.g., total consumption or aggregate share index), and

π = market risk premium.

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[8] For example, Gollier (2011) estimated vastly different systematic risk profiles on the sector level for France. The sectors relevant to public investments, i.e., health, education, energy, and transport, yielded consumption betas of –0.24, 0.11, 0.85, and 1.6, respectively. That is, investments in health and education should be favored over energy and transport investments, since such priorities would reduce the overall macroeconomic risk. Hence, each of these four sectors should have a distinct risk-adjusted discount rate that should be lower than the risk-free discount rate for health and higher than the risk-free discount rate for the other sectors.

[9] Baumstark and Gollier (2014) argued that despite its limited domain of applicability, the Arrow–Lind theorem probably played a crucial role in the development of the public sector in many countries over the previous 40 years. In their opinion, relying on a wrong interpretation of the theorem, some lobbies have used the results to support investment projects whose expected rates of return were not high enough to compensate for the increased systematic risks imposed on their stakeholders.

In Table 1, eq. (34) is calculated for the years 2018–2021 using data from Hultkrantz et al. (2014) and PwC (2021). Hultkrantz et al. (2014) estimated macro-risk-adjusted discount rates for infrastructure investments in Sweden based on historical data. The estimated betas[10] ranged from 0.82 to 0.97, with a mean of 0.92.

PwC (2021) surveyed expectations of financial market returns in Sweden among various categories of investors. The risk-free, long-term nominal interest rate expectation was 2.4–2.9% over the 2018–2021 period, and I have adopted the Swedish central bank-targeted long-term inflation rate of 2.0% as an estimate of inflation, i.e., the risk-free real rate of return is estimated to be 0.4–0.9%. The market risk premium on the Swedish stock market in PwC (2021) was estimated to be 6.4–7.7% during the same period.

TABLE 1 Estimates of the SDR (r) using eq. (34) with $\beta = 0.92$ from Hultkrantz et al. (2014) and other data from PwC (2021)

Year	r_f	π	r
2018	0.9%	6.4%	6.8%
2019	0.6%	6.8%	6.9%
2020	0.4%	7.7%	7.5%
2021	0.4%	6.7%	6.6%
		Mean	**6.9%**

6 Discussion

This analysis has assumed that individuals are rational in that they are forward looking and consistent in their preferences over the analyzed periods. Consequently, the model predicts that if an additional public investment is financed by income taxes (e.g., on labor income), the effect on total savings will be offset by a reduction in private saving, i.e., the government cannot increase total savings by increasing the amount of public investment. Hence, the welfare-maximizing discount rate for public investment is the market rent.

Are individuals really rational in their saving decisions? It might be farfetched to assume that people consciously think about, for example, road and rail investments when they make their saving decisions. I argue, however, that the following is not as farfetched. Investments in transport infrastructure mean better job matching and increased real estate prices, i.e., higher income growth and higher wealth growth for individuals. Individuals get used to this higher rate of prosperity growth, and expect a better economic situation when they retire, meaning they find private pension saving to be less important. More public investments also mean

[10] Formally, these are Weitzman-style betas (i.e., gammas according to the terminology of Weitzman, 2001), which are not exactly the same as CAPM**.** betas. Hence, the estimation process has been slightly adjusted to ensure that the betas are in the range of 0–1. This adjustment should not have a major influence on the results, and the Weitzman formula is identical to the CCAPM formula in period 1.

more tax in the first period, and hence a tighter economic situation in the first period, meaning less money left to save.

However, if individuals are in fact not rational (or are constrained by liquidity restrictions) and do not adjust their saving decisions when public investments increase, and the government would like to increase the welfare of citizens by increasing public investments, it would still be optimal to choose the investments with the highest returns. Under such circumstances, why would the government not choose to make public investments in the financial market if it could earn higher returns there than from traditional public investments (with an equivalent degree of risk)? If one accepts that such investments are an alternative, again the opportunity cost corresponding to the social discount is the pre-tax market return on capital.

A strength of this result is that a welfare-optimal baseline policy is not required for it to hold. What are the main weaknesses of the model? A crucial simplification is that the labor supply is assumed to be exogenous in each period. How big a problem is this likely to be? With an endogenous labor supply, it is easy to show that:

$$u'(c_1) = u'(c_2) \cdot \beta_p \cdot R_p \quad (14)$$

$$c_2 = c_1 \cdot G \quad (17)$$

will still hold, which is the main driver of the result, implying that $r_s{}^* = r_p$ will potentially hold even with an endogenous labor supply. Analyzing this formally in future work would be valuable, as would clarifying the link between the marginal cost of public funds and the SDR in such a model. Other important extensions in future work would be to analyze various special cases of the financing of public investments, for example, by a capital tax (as in Sandmo and Drèze, 1971) or possibly also by borrowing.[11]

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[11] This is not as straightforward, though, since then one would have to justify why financing consumption (i.e., tax decreases) in period 1 by borrowing would not be allowed.

References

Arrow, K. J., & Lind, R. C. (1970). Uncertainty and the evaluation of public investment decisions. *The American Economic Review, 60*(3), 364–378.

Baum, S. (2009). "Description, prescription and the choice of discount rates", Ecological Economics, vol. 69, pp. 197–205.

Baumol, W. J. (1968). "On the Social Rate of Discount", The American Economic Review, 58(4), 788–802. http://www.jstor.org/stable/1815533

Baumstark, L., Gollier, C. (2014). "The relevance and the limits of the Arrow-Lind Theorem", Journal of Natural Resources Policy Research, 6:1, 45-49, DOI: 10.1080/19390459.2013.861160.

Boadway, R., (2006). "Principles of Cost-Benefit Analysis". Public Policy Review, 2 (1).

Bradford, D.F. (1975). "Constraints on government investment opportunities and the choice of discount rate", American Economic Review. 65(5): 887-99

Burgess, D.F (2013). "Reconciling alternative views about the appropriate social discount rate, Journal of Public Economics", Volume 97, Pages 9-17, ISSN 0047-2727, https://doi.org/10.1016/j.jpubeco.2012.08.009.

Burgess, D.F, and Zerbe, R. O. (2011). "Appropriate Discounting for Benefit-Cost Analysis", Journal of Benefit-Cost Analysis: Vol. 2: Iss. 2, Article 2. DOI: 10.2202/2152-2812.1065

Dasgupta, P., and Heal, G. M. (1979). Economic Theory and Exhaustible Resources. Cambridge: Cambridge University Press.

Fleurbaey, M., Zuber, S. (2015). "Discounting, risk and inequality: A general approach", Journal of Public Economics, Volume 128, Pages 34-49, ISSN 0047-2727, https://doi.org/10.1016/j.jpubeco.2015.05.003.

Gollier, C. (2011). Le calcul du risque das les investissements public, Centre d'Analyse Stratégique, Rapports & Documents n 36, http://archives.strategie.gouv.fr/cas/system/files/rapport_36_diffusion.pdf.

Gollier, C. (2014). "Discounting for the long run", American Economic Review: Papers & Proceedings 104(5): 534-537.

Gollier, C., and Hammitt, J.K. (2014). "The long-run discount rate controversy", Annual Review of Resource Economics 6(1): 273–295.

Howard, G., (2013). "Discounting for personal and social payments: Patience for others, impatience for ourselves", Journal of Environmental Economics and Management, Volume 66, Issue 3, Pages 583-597, ISSN 0095-0696, https://doi.org/10.1016/j.jeem.2013.07.001.

Hultkrantz, L., Krüger, N. A., Mantalos, P. (2014). "Risk-adjusted long-term social rates of discount for transportation infrastructure investment", Research in Transportation Economics, Volume 47, Pages 70-81, ISSN 0739-8859, https://doi.org/10.1016/j.retrec.2014.09.020.

Jacobs, B. (2008). "The marginal cost of public funds is one at the optimal tax system", Int Tax Public Finance, 25:883–912 https://doi.org/10.1007/s10797-017-9481-0.

Köbberling, V. (2006). “Strength of preference and cardinal utility”, Economic Theory, vol. 27, pp. 375-391.

Laibson, D. (1997). Golden Eggs and Hyperbolic Discounting. The Quarterly Journal of Economics, 112(2), 443–477. http://www.jstor.org/stable/2951242

Liu, L. (2003). “A marginal cost of funds approach to multi-period project evaluation: Implications for the social discount rate”, Journal of Public Economics, 87: 1707-1718.

Lucas, R. E., Jr. (1978). “Asset Prices in an Exchange Economy”, Econometrica 46 (6): 1429-45.

Mankiw, N.G. (2000). Chapter 8-2: "Saving and Investment in a Small Open Economy", Macroeconomics, Fourth edition, Worth Publishers, New York, ISBN: 1-57259-644-9

Mirrlees, J. (1971). “An exploration in the theory of optimum income taxation”, Review of Economic Studies, 38, 175-208.

Moore, M.A., Boardman, A., Vining, A. (2013a). “More appropriate discounting: the rate of social time preference and the value of the social discount rate”, Journal of Benefit-Cost Analysis; 4(1): 1–16, DOI 10.1515/jbca-2012-0008.

Moore, M., Boardman, A., & Vining, A. (2013b). “The choice of the social discount rate and the opportunity cost of public funds”, Journal of Benefit-Cost Analysis, 4(3), 401-409. doi:10.1515/jbca-2012-0008.

OECD (2018). Cost-Benefit Analysis and the Environment: Further Developments and Policy Use, OECD Publishing, Paris.

Pestieau, P.M. (1974). “Optimal taxation and discount rate for public investment in a growth setting”, Journal of Public Economics, Volume 3, Issue 3, Pages 217-235, ISSN 0047-2727, https://doi.org/10.1016/0047-2727(74)90012-7.

PwC (2021). “Riskpremien på den svenska aktiemarknaden". Retrieved 2022-01-19 from https://www.pwc.se/sv/pdf-reports/corporate-finance/riskpremiestudien-2021.pdf.

Ramsey, F.P. (1928). “A Mathematical Theory of Saving”, Economics Journal, Vol 38, 543-559.

Sandmo, A., Drèze, J. H. (1971). “Discount Rates for Public Investments in Closed and Open Economies”, Economica, New Series, Vol. 38, No. 152, pp. 395-412.

Sellitto, M., Ciaramelli, E., di Pellegrino, G. (2010). “Myopic Discounting of Future Rewards after Medial Orbitofrontal Damage in Humans”, Journal of Neuroscience, 30 (49) 16429-16436; DOI: 10.1523/JNEUROSCI.2516-10.2010.

Sjaastad, L.A. and D.L. Wisecarver (1977). “The social cost of public finance. Journal of Political Economy”, 85(3): 513-547.

Spackman, M. (2020). “Social Discounting and the Cost of Public Funds: A Practitioner’s Perspective”. Journal of Benefit-Cost Analysis, (2), 244-271. doi:10.1017/bca.2020.5.

Weitzman, M. L. (2001). “Gamma Discounting”, American Economic Review vol. 91 (1), pp. 260–71.

Appendix A

A.1 Individual's problem

A representative individual faces the following utility maximization problem during two time periods (subscripts 1,2):

$$\max_{c_t, S_p} U(c_1, c_2) = u(c_1) + u(c_2) \cdot \beta_p \tag{4}$$

s.t.

$$c_1 = \left(1 - \tau_{y,1}\right) \cdot y_1 - S_p + \varepsilon_{p,1}, \tag{5}$$

$$c_2 = \left(1 - \tau_{y,2}\right) \cdot y_2 + R_p \cdot S_p + R_s \cdot S_s + \varepsilon_{p,2}. \tag{6}$$

$R_p = \left(1 + (1 - \tau_K) \cdot r_p\right)$ is the after-tax factor return on private investments.

$R_s = (1 + r_s)$ is the factor return on public investments.

c_t is consumption in time period t, with $0 < c_t < \infty$

β_p is the private utility discount factor (corresponding to the private PRTP), with:

$0 < \beta_p \leq 1.$

y_t is taxable non-capital income (labor income etc.) in time period t, here assumed to be exogenous, with:

$0 < y_t < \infty, y_1 \leq y_2.$

r_p denote the exogenous rate of return on private investments, with:

$0 \leq r_p < \infty.$

r_s denote the exogenous rate of return on public (social) investments, with:

$0 \leq r_s < \infty.$

S_p is the endogenous (private) saving between the two time periods, with:

$-\infty < S_p < \infty.$

S_S is the public (social) investment (saving) per capita between the two time periods, which is exogenous from the individual's point of view, with:

$0 \leq S_s \leq \widehat{S_s}$, where $\widehat{S_s}$ is some exougnous positive number, i.e. corresponding to the maximum available amount of potential projects with return r_s.

$\varepsilon_{p,t}$ is the residual at time period t, i.e., the net for disposal in the analyzed time periods from ingoing capital (in period period 1), (non-taxable) government transfers and outgoing capital (in period 2, i.e., it inludes savings in period 2 to future periods not formally modeled). $\varepsilon_{p,t}$ is treated as exogenous, with:

$-\infty < \varepsilon_{P,t} < \infty$.

$\tau_{y,t}$ denote the tax rate on taxable non-capital income (labor income etc.) in time period t,

with $0 \leq \tau_{y,t} < 1$.

τ_K denote the tax rate on capital (labor income etc.) in time period t,

with $0 \leq \tau_K < 1$.

A key assumption is that $u(c_t)$ a differentiable function and that $u'(c_t)$ is a homogeneous function, but otherwise few assumptions on $u(c_t)$ are needed. For simplicity let us assume the familiar isolelastic utility function (with constant relative risk aversion, CRRA):

$$u(c_t) = \frac{{c_t}^{1-\eta_p} - 1}{1-\eta_p} \qquad (7)$$

$\eta_p \neq 1$

where η_p is the private (behavioral) elasticity of marginal utility of consumption (EMUC). Normally, EMUC is assumed to be positive, but this assumption is not necessary for the following algebraic derivations and results. Please note that if $\eta_p = 1$, then $u(c_t) = \ln(c_t)$.

From eq. (7) follows that:

$$u'(c_t) = {c_t}^{-\eta_p} \qquad (8)$$

Let us now turn to the social planner's problem.

A.2 Social planner's problem

The social planner, i.e., the government, faces the following welfare maximization problem:

$$\max_{S_s,\tau_{y,1},\tau_{y,2}} W(c_1, c_2) = v(c_1) + v(c_2) \cdot \beta_s \tag{9}$$

s.t.

$$\tau_{y,1} \cdot y_1 = S_s + \varepsilon_{s,1}, \tag{10}$$

$$\tau_{y,2} \cdot y_2 + \tau_K \cdot r_p \cdot S_p = \varepsilon_{s,2}. \tag{11}$$

c_1, c_2, S_p are functions of $S_s, \tau_{y,1}, \tau_{y,2}$ (according to the individual equilibrium).

τ_K, r_s are exogenous.

$\varepsilon_{s,t}$ is the (exogenous) government expenditures per capita in each time period t (net of revenues other than from taxation), with:

$-\infty < \varepsilon_{s,t} < \infty.$

β_s is the social utility discount factor, (corresponding to the social PRTP), with

$0 < \beta_s \leq 1.$

Equation (10) and (11) are the public budget restrictions in each time period and defines how public investments are financed. As defined here, no extra public lending is allowed to finance public investments, but these equations imply that the reforms should be budget neutral in each time period, i.e., a balanced budget restriction in each period. For the first time period this implies that public investments are financed with a tax increase on income in the first period (e.g., on labor income). For the second period the implication is that if more public investment leads to lower saving by the individuals, this leads to a lower tax revenue from capital tax and this needs to be compensated by a tax increase on income (e.g., labor income) in the second period.

When it comes to the utility function, again, a key assumption is that $v(c_t)$ is a differentiable function and that $v'(c_t)$ is a homogeneous function (see eq. (25)), but otherwise few assumptions on $v(c_t)$ are needed. Again, for simplicity let:

$$v(c_t) = \frac{{c_t}^{1-\eta_s} - 1}{1-\eta_s} \tag{12}$$

where η_s is the social EMUC.

$$v'(c_t) = {c_t}^{-\eta_s} \tag{13}$$

The problem defined by eqs. (9)-(11) can be rearranged as following:

c_1, c_2 are dependent variables of $\tau_{y,1}$, $\tau_{y,2}$, S_s,

$\tau_{y,1}$, $\tau_{y,2}$ are dependent variables of S_s,

S_s is the only independent variable.

Let us now turn to the equilibria implied by this model.

A.3 Individual's equilibrium

Lagrangian function:

$$\theta(c_1, c_2, S_p, \alpha, \beta) = u(c_1) + u(c_2) \cdot \beta_p + \alpha\left(\left(1 - \tau_{y,1}\right) \cdot y_1 - S_p + \varepsilon_{p,1} - c_1\right) + \gamma\left(\left(1 - \tau_{y,2}\right) \cdot y_2 + R_p \cdot S_p + R_s \cdot S_s + \varepsilon_{p,2} - c_2\right)$$

$$\frac{\partial\theta}{\partial c_1} = u'(c_1) - \alpha = 0$$

$$\frac{\partial\theta}{\partial c_2} = u'(c_2) \cdot \beta_p - \gamma = 0$$

$$\frac{\partial\theta}{\partial S_p} = -\alpha + \gamma \cdot R_p = 0$$

$$\frac{\partial\theta}{\partial \alpha} = \left(1 - \tau_{y,1}\right) \cdot y_1 - S_p + \varepsilon_{p,1} - c_1 = 0$$

$$\frac{\partial\theta}{\partial \beta} = \left(1 - \tau_{y,2}\right) \cdot y_2 + R_p \cdot S_p + R_s \cdot S_s + \varepsilon_{p,2} - c_2 = 0$$

These equations give:

$\alpha = u'(c_1)$

$\gamma = u'(c_2) \cdot \beta_p$

$\alpha = \gamma \cdot R_p$

The two first FOC into the last gives:

$$u'(c_1) = u'(c_2) \cdot \beta_p \cdot \left(1 + (1 - \tau_K) \cdot r_p\right) \quad (14)$$

From restriction we also have:

$$c_1 = \left(1 - \tau_{y,1}\right) \cdot y_1 - S_p + \varepsilon_{p,1}, \quad (15)$$

$$c_2 = \left(1 - \tau_{y,2}\right) \cdot y_2 + R_p \cdot S_p + (1 + r_s) \cdot S_s + \varepsilon_{p,2}, \quad (16)$$

From these equations (14-16) c_1, c_2, S_p can be solved as functions in only $S_s, \tau_{y,1}, \tau_{y,2}$.

Inserting eq. (8) into (14) gives[12]:

$$u'(c_1) = u'(c_2) \cdot \beta_p \cdot R_p$$

$$c_1^{-\eta_p} = c_2^{-\eta_p} \cdot \beta_p \cdot R_p$$

$$c_1 = c_2 \cdot \left[\beta_p \cdot R_p\right]^{-\frac{1}{\eta_p}}$$

$$c_2 = c_1 \cdot G \qquad (17)$$

where G is the growth factor of consumption:

$$G = \left[\beta_p \cdot R_p\right]^{\frac{1}{\eta_p}}, \qquad (18)$$

where $\frac{1}{\eta_p}$ is the elasticity of intertemporal substitution (EIS).

G is exogenous (since all ingoing parameters are exougneous), and hence the proportion between c_1, c_2 is independent of $S_s, \tau_{y,1}, \tau_{y,2}$.

Using eq. (17) c_1, c_2, S_p can be solved as functions in only $S_s, \tau_{y,1}, \tau_{y,2}$ as follows:

Eq. (15) and eq. (16) into eq. (14):

Eq. (15):

$$c_1 = \left(1 - \tau_{y,1}\right) \cdot y_1 - S_p + \varepsilon_{p,1},$$

eq (16):

$$c_2 = \left(1 - \tau_{y,2}\right) \cdot y_2 + R_p \cdot S_p + R_s \cdot S_s + \varepsilon_{p,2}.$$

Eq. (14):

[12] It is easy to see that a key assumption for (14) to take this form is that $u'(c_t)$ is a homogeneous function.

$$\left(1-\tau_{y,1}\right)\cdot y_1 - S_p + \varepsilon_{p,1} = \left[\left(1-\tau_{y,2}\right)\cdot y_2 + R_p\cdot S_p + R_s\cdot S_s + \varepsilon_{p,2}\right]\cdot B^{-1}$$

$$-S_p - R_p\cdot S_p\cdot G^{-1} = \left[\left(1-\tau_{y,2}\right)\cdot y_2 + R_s\cdot S_s + \varepsilon_{p,2}\right]\cdot B^{-1} - \left(1-\tau_{y,1}\right)\cdot y_1 - \varepsilon_{p,1}$$

$$-S_p\left\{1 + R_p\cdot G^{-1}\right\} = \left[\left(1-\tau_{y,2}\right)\cdot y_2 + R_s\cdot S_s + \varepsilon_{p,2}\right]\cdot G^{-1} - \left(1-\tau_{y,1}\right)\cdot y_1 - \varepsilon_{p,1}$$

$$S_p = \frac{-\left[\left(1-\tau_{y,2}\right)\cdot y_2 + R_s\cdot S_s + \varepsilon_{p,2}\right]\cdot G^{-1} + \left(1-\tau_{y,1}\right)\cdot y_1 + \varepsilon_{p,1}}{1 + R_p\cdot G^{-1}}$$

$$S_p\left(S_s, \tau_{y,1}, \tau_{y,2}\right) = \frac{-\left[\left(1-\tau_{y,2}\right)\cdot y_2 + R_S\cdot S_S + \varepsilon_{p,2}\right] + \left[\left(1-\tau_{y,1}\right)\cdot y_1 + \varepsilon_{p,1}\right]\cdot G}{G + R_p} \qquad (19)$$

Eq (16) into eq. (15):

$$c_1 = \left(1-\tau_{y,1}\right)\cdot y_1 - S_p + \varepsilon_{p,1}$$

$$c_1 = \left(1-\tau_{y,1}\right)\cdot w_1 L_1 - \frac{-\left[\left(1-\tau_{y,2}\right)\cdot y_2 + R_s\cdot S_s + \varepsilon_{p,2}\right] + \left[\left(1-\tau_{y,1}\right)\cdot y_1 + \varepsilon_{p,1}\right]\cdot G}{G + R_p} + \varepsilon_{P,1}$$

$$c_1\left(S_s, \tau_{y,1}, \tau_{y,2}\right) = \left(1-\tau_{y,1}\right)\cdot w_1 L_1 + \frac{\left[\left(1-\tau_{y,2}\right)\cdot y_2 + R_S\cdot S_S + \varepsilon_{p,2}\right] - \left[\left(1-\tau_{y,1}\right)\cdot y_1 + \varepsilon_{p,1}\right]\cdot G}{G + R_p} + \varepsilon_{p,1} \qquad (20)$$

Eq (16) into eq. (16):

$$c_2\left(S_s, \tau_{y,1}, \tau_{y,2}\right) = \left(1-\tau_{y,2}\right)\cdot y_2 + + \frac{-\left[\left(1-\tau_{y,2}\right)\cdot y_2 + R_S\cdot S_S + \varepsilon_{p,2}\right] + \left[\left(1-\tau_{y,1}\right)\cdot y_1 + \varepsilon_{p,1}\right]\cdot G}{\frac{G}{R_p} + 1} + R_s\cdot S_s + \varepsilon_{p,2} \qquad (21)$$

A.4 The welfare maximizing discount rate

To find the social discount rate $r_s{}^*$ that leads to welfare maximizing choices when making public investment decisions, we don't have to solve the optimization problem defined in eqs. (6) - (9). The welfare maximizing social discount rate $r_s{}^*$ is here defined as the lowest return on public investments r_s that make an additional (marginal) public investment increase welfare. Such lower limit of r_s is the r_s that make an additional public investment not have any effect on welfare. I.e.,

$$r_s{}^* = r_s$$

where r_s solves:

$$\frac{dW}{dS_s} = 0, \qquad (22)$$

given that the constraints are fulfilled.

$$\frac{dW}{dS_s} = v'(c_1) \cdot \frac{dc_1}{dS_s} + v'(c_2) \cdot \frac{dc_2}{dS_s} \cdot \beta_s \qquad (23)$$

From individual's equilibrium (eq. 17) we have:

$$c_2 = c_1 \cdot G$$

Assuming that c_1, c_2 are continuous and differentiable functions of S_s (on the relevant interval), and that the above equation (eq. 17) holds for all S_s, differentiation is possible:

$$\frac{dc_2}{dS_s} = \frac{dc_1}{dS_s} \cdot G \qquad (24)$$

We also have $v'(c_t) = c_t{}^{-\eta_s}$ from eq. (13).

Inserting eqs. (13), (17) and (24) into eq. (23)[13] gives the new the equilibrium condition:

$$\frac{dc_1}{dS_s} \cdot (1 + G^{1-\eta_s} \cdot \beta_s) = 0 \qquad (25)$$

As G and β_s are positive (by the definition of the ingoing parameters) this gives: $(1 + G^{1-\eta_s} \cdot \beta_s) \neq 0$, and hence eq. (25) implies:

[13] Again, it is easy to see that a key assumption for (20) to take this form is that $v'(c_t)$ is a homogeneous function.

$$\frac{dc_1}{dS_s} = 0 \tag{26}$$

I.e., we only need to find $\frac{dc_1}{dS_s}$.

Rearrangement of eq. (20) for easier derivation:

$$c_1 = (1-\tau_{y,1}) \cdot y_1 + \frac{(1-\tau_{y,2}) \cdot y_2 + R_s \cdot S_s + \varepsilon_{p,2} - [(1-\tau_{y,1}) \cdot y_1 + \varepsilon_{p,1}] \cdot G}{G + R_p} + \varepsilon_{P,1}$$

$$c_1 = R_s \cdot S_s \cdot \frac{1}{G+R_p} + (1-\tau_{y,1}) \cdot y_1 + \frac{-(1-\tau_{y,1}) \cdot y_1 \cdot G}{G+R_p} + \frac{(1-\tau_{y,2}) \cdot y_2}{G+R_p} + \varepsilon_{P,1} + \frac{\varepsilon_{p,2} - \varepsilon_{p,1} \cdot G}{G+R_p}$$

$$c_1(S_s, \tau_{y,1}, \tau_{y,2}) = \frac{1}{G+R_p} \cdot R_s \cdot S_s + y_1 \cdot \left\{1 - \frac{G}{G+R_p}\right\} \cdot (1-\tau_{y,1}) + \frac{y_2}{G+R_p} \cdot (1-\tau_{y,2}) + + \varepsilon_{P,1} + \frac{\varepsilon_{p,2} - \varepsilon_{p,1} \cdot G}{G+R_p}$$

$$c_1(S_s, \tau_{y,1}, \tau_{y,2}) = \frac{1}{G+R_p} \cdot R_s \cdot S_s + y_1 \cdot \left\{\frac{R_p}{G+R_p}\right\} \cdot (1-\tau_{y,1}) + \frac{y_2}{G+R_p} \cdot (1-\tau_{y,2}) + + \varepsilon_{P,1} + \frac{\varepsilon_{p,2} - \varepsilon_{p,1} \cdot G}{G+R_p}$$

$$\frac{dc_1}{dS_s} = \frac{1}{G+R_p} \cdot R_s - y_1 \cdot \frac{R_p}{G+R_p} \cdot \frac{d\tau_{y,1}}{dS_s} - \frac{y_2}{G+R_p} \cdot \frac{d\tau_{y,2}}{dS_s}$$

$$\frac{dc_1}{dS_s} = \frac{1}{G+R_p} \cdot \left[R_s - y_1 \cdot R_p \cdot \frac{d\tau_{y,1}}{dS_s} - y_2 \cdot \frac{d\tau_{y,2}}{dS_s}\right] \tag{27}$$

Now let us turn to the derivative of $\tau_{y,1}$. The restrictions (10) and (11) are conveniently on the form that make $\tau_{y,1}$ a function of only S_s, and $\tau_{y,2}$ may be solved as a more complex function of S_s:

Eq (10):

$$\tau_{y,1} \cdot y_1 = S_s + \varepsilon_{s,1}$$

$$\tau_{y,1}(S_s) = \frac{S_s + \varepsilon_{s,1}}{y_1} \tag{28}$$

Now let us turn to the derivative of $\tau_{y,2}$.

Eq. (19) into eq. (11):

$$\tau_{y,2}(S_s) = \frac{\varepsilon_{S,2} \cdot (G+R_p) + \tau_K \cdot r_p \cdot \{y_2 + R_S \cdot S_S + \varepsilon_{P,2} - [y_1 - S_S - \varepsilon_{S,1} + \varepsilon_{p,1}] \cdot G\}}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \tag{29}$$

$$\tau_{y,2}(S_s) = \frac{\tau_K \cdot r_p \cdot R_s}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \cdot S_s + \frac{\tau_K \cdot r_p \cdot G}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \cdot [S_s - y_1 + \varepsilon_{S,1} - \varepsilon_{P,1}] + \frac{\varepsilon_{S,2} \cdot G + R_p + \tau_K \cdot r_p \cdot \{y_2 + \varepsilon_{P,2}\}}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}}$$

$$\tau_{y,2}(S_s) = \frac{\tau_K \cdot r_p \cdot (R_s + G)}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \cdot S_s + \frac{\tau_K \cdot r_p \cdot G}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \cdot [-y_1 + \varepsilon_{S,1} - \varepsilon_{P,1}] + \frac{\varepsilon_{S,2} \cdot G + R_p + \tau_K \cdot r_p \cdot \{y_2 + \varepsilon_{P,2}\}}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}}$$

$$\tau_{y,2}(S_s) = \frac{\tau_K \cdot r_p \cdot (R_s + G)}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \cdot S_s + \frac{\tau_K \cdot r_p \cdot G}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \cdot [\varepsilon_{S,2} - y_1 + \varepsilon_{S,1} - \varepsilon_{P,1}] + \frac{R_p + \tau_K \cdot r_p \cdot \{y_2 + \varepsilon_{P,2}\}}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}}$$

Differentiation of eqs. (29) and (30) gives:

$$\frac{d\tau_{y,1}}{dS_s} = \frac{1}{y_1} \tag{30}$$

$$\frac{d\tau_{y,2}}{dS_s} = \frac{\tau_K \cdot r_p \cdot (R_s + G)}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}} \tag{31}$$

(31) and (32) into (27):

$$\frac{dc_1}{dS_s} = \frac{1}{G + R_p} \cdot \left[R_s - y_1 \cdot R_p \cdot \frac{d\tau_{y,1}}{dS_s} - y_2 \cdot \frac{d\tau_{y,2}}{dS_s}\right]$$

$$\frac{dc_1}{dS_s} = \frac{1}{G + R_p} \cdot \left[R_s - y_1 \cdot R_p \cdot \frac{1}{y_1} - y_2 \cdot \frac{\tau_K \cdot r_p \cdot (R_s + G)}{y_2 \cdot \{G + R_p + \tau_K \cdot r_p\}}\right]$$

$$\frac{dc_1}{dS_s} = \frac{1}{G + R_p} \cdot \left[R_s - R_p - \frac{\tau_K \cdot r_p \cdot (R_s + G)}{\{G + R_p + \tau_K \cdot r_p\}}\right]$$

$$\frac{dc_1}{dS_s} = \frac{1}{G + R_p} \cdot \left[R_s \cdot \left(1 - \frac{\tau_K \cdot r_p}{\{G + R_p + \tau_K \cdot r_p\}}\right) - R_p - G \cdot \left(\frac{\tau_K \cdot r_p}{\{G + R_p + \tau_K \cdot r_p\}}\right)\right]$$

$$\frac{dc_1}{dS_s} = \frac{1}{G + R_p} \cdot \left[R_s \cdot \left(\frac{G + R_p}{\{G + R_p + \tau_K \cdot r_p\}}\right) - R_p - G \cdot \left(\frac{\tau_K \cdot r_p}{\{G + R_p + \tau_K \cdot r_p\}}\right)\right]$$

$$\frac{dc_1}{dS_s} = R_s \cdot \left(\frac{1}{\{G + R_p + \tau_K \cdot r_p\}}\right) - \frac{1}{G + R_p} \cdot \left[R_p + G \cdot \left(\frac{\tau_K \cdot r_p}{\{G + R_p + \tau_K \cdot r_p\}}\right)\right]$$

$$\frac{dc_1}{dS_s} = R_s \cdot \left(\frac{1}{\{G + R_p + \tau_K \cdot r_p\}}\right) - \frac{1}{G + R_p} \cdot \left[G + R_p - G + G \cdot \left(\frac{\tau_K \cdot r_p}{\{G + R_p + \tau_K \cdot r_p\}}\right)\right]$$

$$\frac{dc_1}{dS_s} = R_s \cdot \left(\frac{1}{\{G + R_p + \tau_K \cdot r_p\}}\right) - 1 - \frac{1}{G + R_p} \cdot \left[-G + G \cdot \left(\frac{\tau_K \cdot r_p}{\{G + R_p + \tau_K \cdot r_p\}}\right)\right]$$

$$\frac{dc_1}{dS_s} = R_s \cdot \left(\frac{1}{\{G + R_p + \tau_K \cdot r_p\}}\right) - 1 - \frac{G}{G + R_p} \cdot \left[-1 + \left(\frac{\tau_K \cdot r_p}{\{G + R_p + \tau_K \cdot r_p\}}\right)\right]$$

$$\frac{dc_1}{dS_s} = R_s \cdot \left(\frac{1}{\{G + R_p + \tau_K \cdot r_p\}}\right) - 1 + \frac{G}{G + R_p} \cdot \left[1 - \left(\frac{\tau_K \cdot r_p}{\{G + R_p + \tau_K \cdot r_p\}}\right)\right]$$

$$\frac{dc_1}{dS_s} = R_s \cdot \left(\frac{1}{\{G + R_p + \tau_K \cdot r_p\}}\right) - 1 + \frac{G}{G + R_p} \cdot \left(\frac{G + R_p}{\{G + R_p + \tau_K \cdot r_p\}}\right)$$

$$\frac{dc_1}{dS_s} = R_s \cdot \left(\frac{1}{\{G + R_p + \tau_K \cdot r_p\}}\right) - 1 + G \cdot \left(\frac{1}{\{G + R_p + \tau_K \cdot r_p\}}\right)$$

$$\frac{dc_1}{dS_s} = \frac{R_s - \{G + R_p + \tau_K \cdot r_p\} + G}{\{G + R_p + \tau_K \cdot r_p\}}$$

$$\frac{dc_1}{dS_s} = \frac{R_s - \{R_p + \tau_K \cdot r_p\}}{\{G + R_p + \tau_K \cdot r_p\}}$$

$$R_p = \left(1 + (1 - \tau_K) \cdot r_p\right)$$

$$R_s = (1 + r_s)$$

$$R_s = (1 + g)$$

$$\frac{dc_1}{dS_s} = \frac{1 + r_s - \{1 + (1 - \tau_K) \cdot r_p + \tau_K \cdot r_p\}}{\{1 + g + \left(1 + (1 - \tau_K) \cdot r_p\right) + \tau_K \cdot r_p\}}$$

$$\frac{dc_1}{dS_s} = \frac{1 + r_s - \{1 + r_p\}}{\{1 + g + 1 + r_p\}}$$

$$\frac{dc_1}{dS_s} = \frac{r_s - r_p}{2 + g + r_p}$$

Equilibrium condition:

$\frac{dc_1}{dS_s} = \frac{r_s - r_p}{2 + g + r_p} = 0$ (32)

As $1 + g = G = \frac{c_2}{c_1} > 0$ and $r_p \geq 0$ we can conclude that denominator is strictly larger than 0 and hence:

$$r_s - r_p = 0$$

$r_s{}^* = r_p$ (33)