Baserunning - analyzing the sensitivity and economies of scale of the Swedish national freight model system using stochastic production-consumption-matrices

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Abstract: The purpose of the paper is to analyze how sensitive the Swedish national freight model system *Samgods* is to uncertainties in its production-consumption matrices (PC-matrices). This is done by studying how sensitive outputs from one of its key component, the logistics model, are to changes in the PC-matrices. This paper is, to our knowledge, the first attempt to analyze the sensitivity and economies of scale of a national freight transport model using Monte Carlo simulation. The results indicate that the logistics model is able to find new logistics solutions when larger demand volumes are assumed. Freight volumes are calculated to shift to sea transport. If the transport volume increases with one percent, the logistics cost per tonne is on average reduced by about 0.5 percent. Part of the cost reduction comes from increased consolidation of shipments due to larger transport volumes. There is also a positive correlation between total transport demand and the load factor for heavier lorries, trains and larger ships. Without empirical data and further analysis it is difficult to assess the estimated strength of the effect. Furthermore, the analysis indicates that it might be possible to reduce runtimes by removing small transport flows from the PC-matrices without affecting aggregate results too much.

Keywords: sensitivity analysis, large scale freight model, Monte Carlo simulation
JEL Codes: C52, C63, R41, R42

PREFACE

There are indications that input uncertainty is more important for transport forecasts than model uncertainty. Therefore Niclas Krüger and Inge Vierth had the idea to study how the fixed commodity specific production-consumption matrices (PC-matrices) that describe freight transport demand in the Samgods model influence the results in terms of tonne-kilometers per mode, transport costs per tonne-kilometer etc.

Possible types of experiments were discussed together with Gerard de Jong, Rune Karlsson, Magnus Johansson and Jonas Westin who have experience with the Samgods model. The group agreed on four experiments: (1) Stochastic changes to the PC-matrices for various independent regions, (2) Scaling the PC-matrices up and down by the same amount for all cells, (3) Stochastic changes in PC flow for different regions with regional correlation based on a time series model for national and international transports, (4) Removal of small flows from the PC-matrices.

The experiments were prepared by Jonas Westin, Niclas Krüger, Magnus Johansson and Rune Karlsson. The nearly 250 "Samgods model runs" were carried out by Jonas, Rune and Magnus and analyzed using post processing tools developed by Rune and Jonas. Results were analyzed by Jonas and Rune as well as in meetings with the whole project group. Gerard has written most of the text on "Sensitivity analysis in transport". Inge has coordinated the project.

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Stockholm, May 2015

1   INTRODUCTION

The existing version of the Swedish national freight transport model system Samgods comprises of 34 fixed commodity-specific PC-matrices for 464 zones[1] in a given year, a deterministic *logistics model* that minimizes the shippers' annual logistics costs[2,3] and a network model that distributes the supply chains over the transport infrastructure. The PC-matrices are constructed using data from regional accounts, input-output tables, foreign trade statistics, the Swedish Commodity Flow Survey (CFS) as well as different models, i.e. gravity models (Edwards et al., 2008; Edwards, 2008). Since the PC-matrices are estimated for a certain base year, they are uncertain. Due to limited information on regional transport demand, it is very difficult to validate the PC-matrices as such. Transport and traffic forecasts are however calculated in the Samgods model and can be validated for the base year.

1.1   Logistics model

Within the logistics model, the firms' annual logistics costs are minimized, taking into account the tradeoff between transport costs and warehouse costs. Also taken into account is the fact that transport costs per unit can be reduced by using larger vehicle types when transporting goods from one or several shippers. The choice between container and non-container chains is also modelled. By combining predefined transport costs per vehicle kilometer with calculated load factors, transport costs per tonne-kilometer are calculated. Based on these inputs the model determines the shippers' choice from a set of constructed transport chains. It is assumed that the transport companies pass all cost changes to the shippers.

The logistics model comprises in total of 33 vehicle types: five road, eight rail, 19 sea and one air. For sea transports, different types of vessels (container, ro-ro and other vessels) and ferries are included.[4] Different vehicle sizes allows for economies of scale to be modeled. This aspect is especially important for vessels that differ significantly in size (and therefore costs). The capacity of the vessels varies from 1 000 to 250 000 dwt. Transport costs comprise underway costs and transfer costs. The underway costs do in turn comprise of time-based costs and distance-based costs as well as infrastructure fees.[5] The model also takes into account infrastructure restrictions in form of maximum depth for vessels and maximum weight for trucks and trains. Capacity restrictions in terms of number of trains per track and vessels per port are not included in the model version used in this study.[6] Capacity problems in ports are also assumed to be negligible.

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[1] The zones are 290 municipalities in Sweden and 174 larger administrative regions outside Sweden.

[2] The logistics costs comprise of transport costs, order costs and warehouse costs. For an overview of the Samgods logistics model, see: de Jong and Ben-Akiva (2007), Vierth et al. (2009) or Significance (2011).

[3] The development of a stochastic logistics module has started (Abate, Vierth and de Jong, 2014).

[4] Different average speeds are assumed for different vehicle/vessel types, i.e. for container vessels 30–39 km/h, for ro-ro vessels 30 km/h, for other vessels 22–30 km/h.

[5] Infrastructure fees comprise e.g. fairway dues, pilot fees.

[6] Another project is underway to model the limited rail capacity using linear programming.

1.2 Purpose of the paper

The purpose of this paper is to analyze the sensitivity of the Samgods model to uncertainties in the PC-matrices by studying how sensitive the outputs of the logistics model (such as tonne-kilometer by mode and transport cost) are for changes in one of its key inputs: PC-matrices. The tests are limited to the use of different zone-to-zone PC-matrices; the disaggregation from zone-to-zone flows to firm-to-firm flows is kept constant.

The results derived in the paper can for example be used to estimate confidence intervals for transport forecasts where the degree of uncertainty increases compared to the base year. Sensitivity analysis is often used in transport modelling (though infrequently in combination with stochastic variation of the inputs) to obtain uncertainty margins or confidence intervals for the model outputs.

The paper begins with a review of the application of sensitivity analysis in transport.

2 SENSITIVITY ANALYSIS IN TRANSPORT

2.1 Reasons to carry out a sensitivity analysis

There can be several reasons to carry out a sensitivity analysis in the context of a transport model:

- Most often such analyses are done to check the quality of a model. After a model has been estimated, it is often good practice to vary input variables such as transport time and cost by mode and investigate how the outputs of the transport model vary as a result of this. The results of the sensitivity analyses can be expressed in form of elasticities (a percentage change in an output variable divided by a percentage change in an input variable), which can be compared to elasticities based on other data for the same area and/or international literature.[7] If a model's elasticities are not plausible, this may be a reason to try a different model specification.
- Sometimes sensitivity analyses are carried out to get an idea of the possible range of the outputs: instead of a prediction in the form of a central value (point estimate) of some model output variable, policy-makers sometimes want to have an uncertainty margin (e.g. 95 % confidence interval) around the central value for this variable. This makes it possible to select robust policy measures: measures that have desired consequences for all likely outcomes of this output variable.
- A third reason for doing sensitivity analysis, which does not occur often, is for obtaining inputs for a fast and simple model that reproduces the main responses of one or more slow and detailed models. The fast model then is a

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[7] For freight transport, relevant international reviews of elasticity values are Significance and CE Delft (2010) and VTI and Significance (2010).

so-called 'repro-model', and one of the possible forms for this is an 'elasticity-model'.

The motivation for doing sensitivity analysis to changes in the PC-matrices in this project is a combination of the first and the second reason from the list above. The results on the model outputs might be expressed as percentage changes, but elasticities cannot be calculated, because changes to the PC-matrices are not of a simple overall percentage change nature (except when scaling the entire PC-matrices up or down by the same percentage).

2.2 Sensitivity to what?

Sensitivity analysis studies the responses to changes in one or a combination of the following changes:
- Changes in the input variables to the transport model (e.g. changes in fuel price or vehicle loading capacity).
- Changes in the transport model itself:
  - Changes in the specification of the model: different functional forms (e.g. linear versus log-linear or Box-Cox) or a difference in the selection of variables included in the model.
  - Changes in the coefficient (sometimes called: parameter) values of the model: a statistical estimation procedure may be able to find the 'most likely' coefficient values, given the data, but there will be some chance that the true coefficient values differ from those estimated values. The standard deviations of the estimates can be used to define confidence intervals around the estimated coefficients.

In the review in RAND Europe (2005) and de Jong et al. (2007) the two reasons for uncertainty in the transport model outputs are called 'input uncertainty' and 'model uncertainty'. In Monte Carlo simulations with the Dutch National and Regional passenger transport models by these authors, input uncertainty was clearly more important for the uncertainty in the model outputs than model uncertainty (but please note that the latter here only included uncertainty in the coefficient values of the model).

Investigating the sensitivity of logistics model outputs to changes in the PC demand matrices is a special form of sensitivity analysis of input variables. Instead of a single input value, the PC-matrices contain a very large number of values that refer to a base year or some future year. In this paper we stick to the PC-matrices in the base year (in our case 2006) knowing that the matrices are uncertain since they are not based on perfect information and are influenced by different kinds of measurement (and matrix modelling) errors.

2.3 Methods for doing sensitivity analysis

Usually the model inputs are known for the base year, but are uncertain for a future year. The latter uncertainty leads to uncertainty in the model outputs for a future year. But even for the base year, the input variables are in general the result of some estimation and might therefore be uncertain. This is certainly the case for the base year PC-matrices in the Samgods model.

The first distinction in sensitivity testing of input variables is that between:
- Varying one input variable at the time. This can be the basis for calculating elasticities. One can either change autonomous variables such as GDP or the oil price, to find out how large the influence of the external environment is on the variables of interest, or change policy variables, such as the fuel tax, to simulate the impact of potential policy measures.
- Varying all (key) input variables together. The most common procedure for doing this is scenario analysis. A much more uncommon method is a systematic Monte Carlo simulation that produces uncertainty margins for the model outputs on the basis of uncertainty margins in the input variables.

For model coefficients, the same distinction can be made between varying one coefficient at a time or all coefficients together. The latter produces a confidence interval for the model outputs for all uncertainty that is due to using estimated coefficients instead of given true values. Scenario analysis usually does not involve varying the values of the model coefficients, whereas Monte Carlo simulation can be used both for model inputs and model coefficients. Below we will discuss scenario analysis and Monte Carlo simulation in more detail.

2.3.1 Scenario analysis

Scenario analysis was pioneered by Shell and RAND Corporation in the early seventies to investigate the impact that the main external forces together will have on the outcomes of interest. A scenario is a consistent picture of a possible future. It consists of a number of assumptions on the values of input variables that are all part of an overall view of how a system may develop (e.g. a scenario for a situation of increased protectionism versus a scenario of free trade; a green scenario versus an economy-first scenario).

The key to scenario building is determining which external influencing factors are most important for the outcomes of interest (such as tonne- kilometer by mode, or emissions) and which levels these variables might take in a possible future. Only factors that are likely to change and for which changes are likely to have a large influence on the outcomes of interest need to be included in a scenario. To make the scenarios internally consistent, the scenarios need to take into account that several influencing factors can be correlated over time (so for instance a scenario with a high income growth will also have a high consumption).

Scenario analysis has been used in very diverse fields, including energy policy, military strategy-making and economics. In transport research, it has been used in many countries, either building on general-purpose scenarios (e.g. the Dutch

WLO scenarios, CPB et al., 2006), or constructing scenarios specifically for the transport sector, such as the STEPs (Fiorello et al., 2006) and the TRANSVISION (Petersen et al., 2009) scenarios for Europe, or the scenarios in the Flanders Mobility Masterplan (de Jong et al., 2010).

Different scenarios can also contain different assumptions on the (future) zonal distribution of some input variable (such as GDP, population, employment), e.g. a centralization scenario versus a decentralization scenario.

An important element of a scenario analysis is the idea that various scenarios should be tested: a model should be run for at least two, but preferably more possible future states of the world. Scenario studies usually cover between two and five scenarios. If possible, the set of different scenarios tested in a study should cover most of the likely variation in the influencing factors. However, no probabilities are attached to the various scenarios: there is no indication that one scenario is more likely than another, or that all are equally likely. This makes it impossible to derive uncertainty margins for the output variables from scenario analysis. Policy-makers sometimes have revealed a tendency to focus on a 'middle' scenario (e.g. when there are three scenarios in terms of economic growth: low, medium and high growth), but this is not in line with the general idea of scenario analysis.

2.3.2 Monte Carlo simulation

The use of Monte Carlo simulation in the application of transport models means that numbers are drawn from a statistical distribution, such as the normal or uniform distribution, and that the model is run repeatedly for each set of numbers. One might draw random numbers both for input variables and for model coefficients. The outcomes of the various runs may be summarized, using the mean for some model outcome and the standard deviation. To get stable results one needs (depending on the application) at least several dozens of runs, but hundreds or thousands is not exceptional. Because the selection of input variables (or of model coefficients) is done from a known (assumed) distribution, confidence intervals for the model outputs can be calculated.

In RAND Europe (2005), de Jong et al. (2007), Rasouli and Timmermans (2012) and Manzo (2014) reviews are provided of studies in transport that have provided uncertainty margins for transport model outputs. Most of these studies have used Monte Carlo simulation. For quantifying input uncertainty, this is the only approach used in the literature that can produce confidence intervals. For finding a confidence interval for model outputs that is due to variation in the model coefficients, there is a wider variety of methods. Besides Monte Carlo simulation, some studies use the analytic expression of the variance of the model output as a function of the variances of the model coefficients. This is only a feasible alternative if the model is relatively simple (and if proper variances for the parameter estimates are available from the estimation process). Examples of the use of the analytic method can be found in Ben-Akiva and Lerman (1985) and Daly et al. (2012). The method selected for the application to the Dutch national and regional passenger transport models in RAND Europe (2005) and de Jong et al. (2007) was also Monte Carlo simulation for both inputs and coefficients.

The difficult issue in the application of Monte Carlo simulation is how to determine the distribution to draw from and its means, variances and covariances. Ideally one would want to use a multivariate distribution containing all important input variables (and/or model coefficients). In practice, most studies use univariate distributions and assume that the different influencing variables are independent (no correlations) or that the correlation structure is very simple (e.g. by grouping variables into perfectly correlated subsets), see for example the analysis in Westin and Kågeson (2012). A common method for risk analysis is to take the likely total range of variation of an input variable (based on the past variation) and to assume a symmetric triangular distribution that covers this range. But some studies have used multivariate Normal distributions taking account of correlation between input variables (or coefficients). The variance-covariance matrix of the model coefficients can come from the model estimation (though for proper estimation, it may be necessary to use resampling methods, such as Jackknife or Bootstrap).

In RAND Europe (2005) and de Jong et al. (2007), the variance-covariance matrix of the multivariate distribution (from which the random draws were made) for the input variables was determined on the basis of time series analysis. Time series for 1960-2000 were available (annual national data). To remove the effect of cyclical fluctuation, which is undesirable in long term forecasting, this study did not use ordinary year-to-year growth rates, but calculated 20-year moving averages for these. The variance-covariance matrix then referred to these moving averages. For the model coefficients, the variance-covariance matrix was based on the estimation results.

There is also an application to Sweden that uses the Swedish national passenger transport model (SAMPERS). Beser Hugosson (2004, 2005) used the Bootstrap method on the disaggregate mode-destination models within SAMPERS and repeated model application to quantify model (coefficient) uncertainty. The outputs were 95 % confidence intervals for total demand, car demand on specific OD relations, flows on specific road links and railway lines and values of time.

Another interesting question is whether there will be propagation of errors: especially when a number of models are used sequentially, errors in the inputs can lead to bigger errors in the model outputs (reinforcing initial deviations), but also to smaller output errors (equilibrium mechanisms). This was studied by Zhao and Kockelman (2002).

2.4   Comparison of forecasts and outturns

The uncertainty margins of the model outputs discussed above all refer to intervals that are calculated (many years) before the forecasting horizon is reached. But after having reached the future year to which the model predictions refer, the practical outturn can be compared with the original model forecast. This can also be very instructive for the confidence intervals that were produced as part of the modelling/ forecasting process on the basis of information from the past, using the methods discussed above.

A number of studies has been looking at the issue of actual versus predicted outputs (especially link flows) in recent years. Flyvbjerg et al. (2006) have found evidence of optimism bias (the mean of the model forecasts is significantly higher than the mean outturn) for public transport projects, but not for toll-free road projects. Bain (2009) did find optimism bias for privately financed toll road projects. For both road and public transport projects, large differences between model predictions and outturns were found. These error margins are larger than what transportation planning professions would expect (see Bain, 2011), and sometimes also larger than the predicted confidence intervals that take both model and input uncertainty into account. Even the most complete studies on uncertainty margins apparently do not fully include all sources of error. From a compilation of Swedish passenger and freight transport forecasts between 1975 and 2005; one conclusion is it has been difficult to predict deviations from historical trends (Vierth, 2005).

3 EXPERIMENTS CARRIED OUT FOR SENSITIVITY ANALYSIS TO CHANGES IN THE PC-MATRICES

The sensitivity of the output of a logistics model to changes in the commodity specific PC-matrices, is a special form of sensitivity analysis to changes in the input variables. In this paper we perform four different tests to analyze the sensitivity of the logistics model to changes in the PC-matrices.

3.1.1 Experiments carried out

First we analyze the effect of random perturbations on the PC-matrices. In the second test we compare the first test to a situation where the PC-matrices are uniformly scaled. The purpose of the first two tests is to create a reference frame for the third test where we use a stochastic model to estimate counterfactual base matrices based on time series data over the economic development in Sweden during the last decades. Last we perform a test where we remove small flows in the PC-matrices.

1) *Stochastic changes to the commodity specific PC-matrices for various independent regions.* In the first test, the PC flows representing freight transport demand in, to and from Sweden are perturbed randomly assuming that individual zone-to-zone and firm-to-firm relationships are independent of each other. The method and the outcomes from using the perturbed matrices are described in section 3.2.

2) *Scaling the PC-matrices up and down by the same amount for all cells.* In this test the perturbations were applied deterministically to the elements in the PC-matrices. This kind of sensitivity analysis can shed light on the firms' possibility to exploit economies of scale: if transport demand increases by say 10 percent, will the transport costs and logistics costs also change by 10 percent or less. Increased transport demand can also effect the level of consolidation. The results of the scaling experiment are discussed in section 3.3.

3) *Correlated PC flows for different regions based on a time series model for national and international transports.* The first two experiments on changing the PC matrix inputs to the logistics model can be seen as two extremes: fully independent PC flows or fully dependent. Neither case is realistic. In the real world, the changes in PC flows will most likely be partly correlated since many goods flows are dependent on business cycles (fast growth-recession-depression, etc.) and on structural economic trends. The observed correlation will not be perfect however, since different sectors (distributed unevenly over space) of the economy show different cycles and trends: some will be leading, some lagging, some will be less affected by the business cycle (or even move counter-cyclical) and some will be very pro-cyclical. In the third test, we assume positive correlations between the PC flows for different regions that are somewhere in between zero and perfect correlation. The assumptions of correlation between PC flows are as much as possible based on observed regularities in international trade flows and gross regional product (GRP)[8] over the past decades (from annual time series data). However, some of these data were itself estimated using models, and we (have to) use gravity models to relate GRP changes to domestic interregional PC flows.

   Based on random draws from a multivariate distribution and based on time series simulation of the trends leading up to the base year, we calculate counterfactual PC-matrices. It would have been equally interesting to study alternative PC-matrices that we might have had as inputs if we had encountered different measurement and modelling errors, but we have no information about the magnitude of these errors that can be regarded as sufficiently trustworthy as input for sensitivity analyses. Section 3.4 contains a description of how, using time series data and models, we determined the multivariate distributions (with correlations) that were used to take random draws of GRP in the Monte Carlo simulation. These draws were used as inputs for a gravity model to calculate inter-regional PC flows. We also determined the possible variation in international PC flows on the basis of time series data on international trade. After that, a large number of runs with the logistics model were carried out for these alternative commodity specific PC-matrices inputs combining the domestic (interregional) and international inputs.

4) *Removal of small flows from the PC-matrices.* A special kind of experiment concerns the deterministic removal of small flows from the PC-matrices. This was done to study possibilities for shortening the run time of the model, by using fewer cells that have to be processed. The outcomes, not only in terms of transport volumes, but also in terms of model run time are analyzed in section 3.5. If removing all flows below a certain size would lead to a large reduction in runtime without losing much precision in the model outcomes, then a user might decide to remove these small PC flows for all later runs to save runtime.

   The main objective in the sensitivity analysis is to analyze the effect on the tonne-kilometer per mode, modal share and the logistics cost (in million SEK) when the PC-matrices are perturbed.

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[8] The regions used for this in Sweden are the counties; Sweden has 21 counties.

3.1.2 Point of reference

As a reference point, the PC-matrices for 2006 and tonne-kilometers for the different modes in the Samgods base scenario are used. We used Samgods model version 2012-09-12 running on Cube 6.0. The total number of tonne-kilometers and modal shares in Sweden are shown in Table 1. It has to be stressed that these outputs from the Samgods model are not fully in line with observations from the official transport statistics and traffic counts since the model is not fully calibrated.

Table 1: Freight flows in, to and from Sweden: Transport performance (1000 tonne-kilometer) and modal shares in percent in the base scenario.

	ROAD	RAIL	SEA	TOTAL
MILLION TONNE-KILOMETER IN SWEDEN	52.649	22.183	45.699	120.531
MODAL SHARES (PERCENT)	43.7%	18.4%	37.9%	
MILLION TONNE-KILOMETER IN, TO AND FROM SWEDEN	97.542	37.783	539.223	676.732[9]
MODAL SHARES (PERCENT)	14.4%	5.6%	79.7%	

The total logistics costs in the base scenario is around SEK 360,000 million. The total cost is divided into three cost components; order cost, holding cost and transport cost as shown in Table 2.

Table 2: Cost components in the base scenario for freight flows in, to and from Sweden.

	TOTAL LOGISTICS COST	COST SHARE
ORDER COST (MILLION SEK)	51 000	14%
HOLDING COST (MILLION SEK)	107 300	30%
TRANSPORT COST (MILLION SEK)	205 300	56%
TOTAL COST (MILLION SEK)	363 600	100%

3.2 Stochastic changes to the PC-matrices for independent regions

In the first experiment, we analyze the general sensitivity of the model to random perturbations in the PC-matrices. All elements in the PC-matrices are perturbed by multiplying the values with scale factors uniformly distributed in the interval [0.8, 1.2]. The scale factor for each element in the PC-matrices is generated independently of each other. Four scenarios are analyzed this way and the total number of tonne-kilometers and modal shares for each transport mode are computed. The coefficient of variation (the standard deviation divided by the mean) for the different transport modes are shown in Table 3.

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[9] The base scenario also has 2.184 million tonne-kilometer of international air freight transport corresponding to a modal share of 0.3 percent. The model does not include air transport for domestic transport in Sweden.

Table 3: Coefficient of variation for tonne-kilometers and modal shares in, to and from Sweden with fully randomly perturbed PC-matrices.

RELATIVE DEVIATION	ROAD	RAIL	SEA	AIR	TOTAL
TONNE-KILOMETER (PERCENT)	0.53%	1.54%	0.33%	0.63%	0.38%

Although the perturbations in the PC-matrices are in the order of up to 20 percent, the variation in aggregated tonne-kilometer is relatively small. Translated into modal shares, the standard deviation of the mode shares aggregated over all commodities is less than 0.07 percent for all transport modes indicating that random perturbations of the elements in the PC-matrices only have a small effect on the aggregate modal shares.

Since each PC matrix is made up of centroids for 290 municipalities within Sweden and 174 regions abroad, and because of the large number of independent perturbations, the changes cancel each other out to a large extent. However, the assumption that the perturbations of the matrix elements are independent is obviously not realistic.

3.3   Scaling the entire PC-matrices up and down by the same amount

In the first experiment we analyzed the situation with completely independent perturbations. In the next experiment we analyze an opposite situation where the changes in the PC-matrices are perfectly correlated. The perturbations of the PC flows are hence deterministic. To do this we analyze six scenarios where the total transport volume in each PC matrix is scaled from -20% to +20% compared to the base scenario.

The modal shares for the assumed PC-demand volumes are shown in Table 4. From the table we see that when the total transport demand increases, the modal share of sea transport increases at the expense of mostly road and to lesser part rail. The modal share for air transport is unaffected by the change in total transport demand volume.

Table 4: Modal shares as a function of the total transport volume.

PC-DEMAND VOLUME IN, TO AND FROM SWEDEN	ROAD	RAIL	SEA	AIR
-20%	14,9%	5,8%	79,0%	0,32%
-10%	14,6%	5,6%	79,5%	0,32%
0%	14,4%	5,6%	79,7%	0,32%
10%	14,1%	5,5%	80,1%	0,32%
20%	14,1%	5,6%	80,0%	0,32%

The changes in modal shares (in percent) for the total number of tonne-kilometers for transports all in, to and from Sweden as a function of the change in total freight volumes are shown in Figure 1.
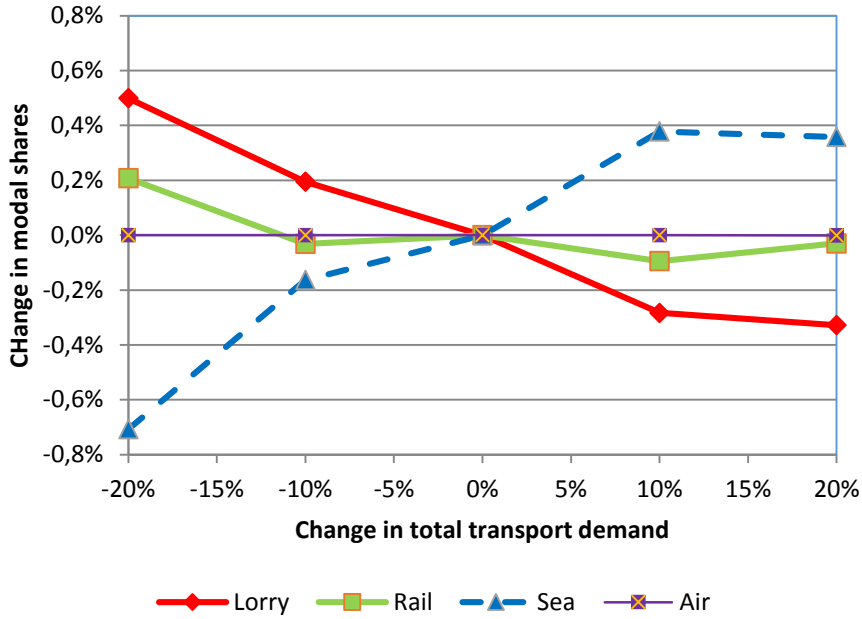
Figure 1: Change in modal shares in, to and from Sweden as a function of change in total transport volume.

The diagram in Figure 2 indicates that increased transport volumes lead to lower average transport and logistics costs for all commodities and trip lengths, as possibilities to exploit economies of scale and to consolidate are improved. To analyze this further we also study the economies of scale in the model by comparing how the average costs (total/transport/order/holding cost divided by total transport volume) is affected by the total transport volume.
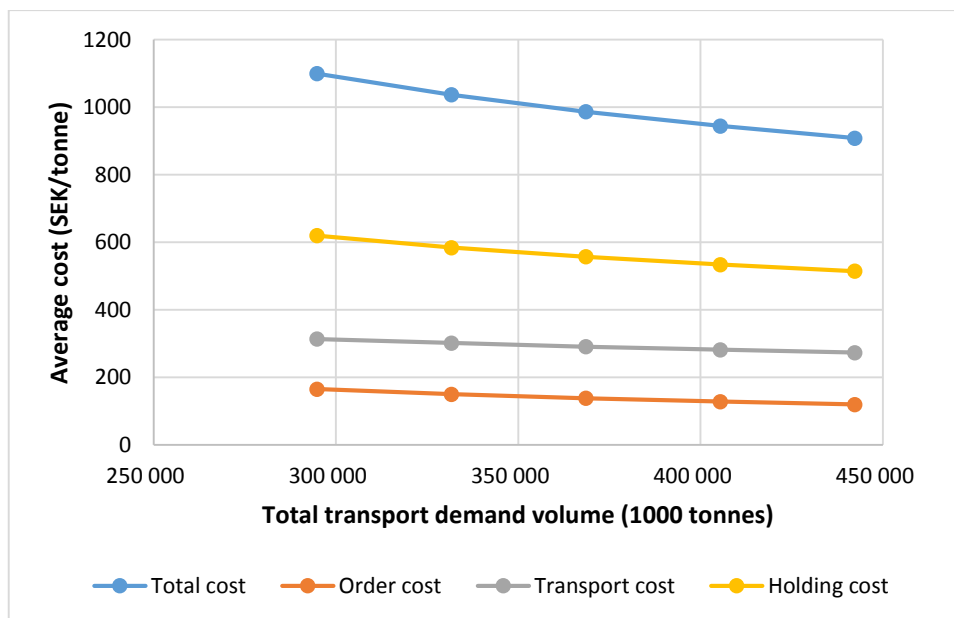

Figure 2: Average cost (SEK/tonne) as a function of total transport demand (1000 tonnes) showing the economy of scale in the Samgods-model to total transport volume.

The figure shows that the average cost per transported tonne is decreasing when the total freight transport demand grows. The effect is strongest for the transport cost whereas the average holding and order cost are less sensitive to the total

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transport volume. To analyze the economies of scale we estimate a simple regression model. Let

$$\left(\frac{AC_i}{AC_0}\right) = \left(\frac{V_i}{V_0}\right)^M \tag{1}$$

where AC_0 is the average total cost per tonne in the base scenario, AC_i is the average total cost in scenario i, V_0 is freight demand volume in tonne for the base case scenario and V_i is the freight demand volume for scenario i. Taking the natural logarithms we formulate a standard regression model

$$\log\left(\frac{AC_i}{AC_0}\right) = M \cdot \log\left(\frac{V_i}{V_0}\right) + \varepsilon_i \tag{2}$$

where M is the scale parameter and ε_i is a normal distributed error term and estimate the parameter M using OLS. For all commodities combined we get a scale factor M is equal to -0.47 which implies that if the freight volume increases with one percent, the average cost decreases with 0.47 percent. Separate estimates of the scale parameters for the different commodities are shown in Table 5.

Table 5: Estimated scale parameter for the different commodities.

COMMODITY	SCALE M	COMMODITY	SCALE M	COMMODITY	SCALE M
1 CEREALS	-0.52	13 Crude petroleum	-0.086	25 Transport equipment	-0.43
2 VEGETABLES	-0.46	14 Petroleum products	-0.35	26 Manufactures of metal	-0.62
3 LIVE ANIMALS	-0.31	15 Iron ore	-0.12	27 Glass, ceramic products	-0.66
4 SUGAR BEET	-0.39	16 Ores and waste	-0.33	28 Paper, paperboard	-0.31
5 PULPWOOD	-0.25	17 Metal products	-0.31	29 Leather, clothing	-0.61
6 WOOD SQUARED	-0.39	18 Cement, lime	-0.40	31 Timber for sawmill	-0.27
7 WOOD CHIPS	-0.51	19 Earth, sand	-0.25	32 Machinery	-0.50
8 OTHER WOOD	-0.83	20 Minerals	-0.27	33 Paper manufactures	-0.60
9 TEXTILES	-0.65	21 Fertilizers	-0.32	34 Wrapping material	-0.65
10 FOODSTUFF	-0.61	22 Coal chemicals	-0.39	35 Air freight	-0.43
11 OIL SEEDS	-0.26	23 Chemicals	-0.37	All	-0.47
12 MINERAL FUELS	-0.21	24 Paper pulp	-0.21		

We find the economies of scale to be relatively low for crude petroleum (that is transported by sea), iron ores, metal waste and building materials and relatively high for food and manufactured products.

3.4 Correlated PC flows for different regions based on a time series model for national and international transports

In the third experiment we use time series data for the Swedish Gross Regional Product (GRP) at county level (Olsson Spjut, 2010) and data on Swedish import and export between 1969 and 2006 (SCB, 2012). Based on the time series data we estimate a multivariate distribution function using a Geometric Brownian

Motion (GBM) model.[10] From the multivariate distribution we then take random draws of the Gross Regional Product in each county and import and export volumes for 2006.

One can regard the materialized GRP-time series as one realization of a stochastic process that the GRP follows, from which we can draw inferences with regard to the underlying process. Whereas a stochastic variable is a random number, a stochastic process is a random path or trajectory. We can see a stochastic process as a collection of stochastic variables for each point of time t and we can therefore define a stochastic process by probability distributions for each point in time.

By estimating the average of GRP-growth and the standard deviation of GRP-growth we can simulate a range of alternative GRP-histories. Each sample path represents a different alternative history, which is equally likely to happen as the one observed. Each of these simulated GRP paths is extremely unlikely to be representative when seen one by one, but by simulating many paths we get a probability distribution for GRP at a certain point of time that makes it possible to calculate expected values for GRP.

The starting point for our analysis is the Geometric Brownian Motion (GBM), which combines an expected growth trend with a random term:

$$\Delta y = y_{t+1} - y_t = \mu y_t \Delta t + \sigma y_t \varepsilon \sqrt{\Delta t}$$

where y is the annual GRP-level, t stands for time, μ is the drift rate showing the expected percentage growth in GRP, σ is the volatility of growth measured as standard deviation in percent and ε represents a draw from the standard normal distribution. The equation is the discrete time version of a stochastic differential equation. It can be shown that the explicit solution for the GBM is:

$$y_t = y_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma z_t} \quad \text{where} \quad z_t = \sum_{i=0}^{t} \varepsilon_i \sqrt{\Delta t}$$

The explicit solution shows that the GBM combines an exponential growth process with a multiplicative random process that makes the simulated paths to deviate from the expected trend growth path. Even a small volatility will have a major impact on the range of possible future outcomes; the range of possible outcomes grow with the square root of time. Moreover, the assumption underlying the GBM is that growth for any short period is randomly distributed, which in turn implies that future GRP-outcomes are log-normally distributed with a positive skew and bounded below by zero. The combination of exponential growth and independent (unforeseeable) random shocks create simulated sample paths that resemble in structure what we observe for GDP-growth over different time periods and countries. Since the GBM depends on only the 2 parameters μ and σ, it is easy to estimate inputs for the simulation based on available historical data. It is also straightforward to generate simulations for

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[10] The model is estimated using the gbm-function in Matlab 2013b and takes time series data over Swedish import and export (by country) and gross regional product (GRP by county) as input.

several correlated stochastic processes by imposing a covariance matrix for the random shocks ε.

Based on the random draws we calculate weights which we then use for rescaling the inter-regional PC flows in the Samgods model. For import between country a and Sweden we set the weight to

$$w_a^i = \frac{I_a^i}{I_a^0}$$

where I_a^0 is the actual value of import from country a in the year 2006 and I_a^i is the random draw of the same variable from the GBM-model. The weights for export are created in a similar way. For domestic transport we calculate weights for transport volumes between pairwise counties a and b using a gravity type model

$$w_{ab}^i = \frac{GDP_a^i \cdot GDP_b^i}{GDP_a^0 \cdot GDP_b^0}$$

where GDP_a^0 is the actual value on the gross regional product in county a and GDP_a^i is the random draw of the same variable.
To simplify the simulation, the same set of weights are used for all commodity groups. We then run the Samgods model with the rescaled PC-matrices and record a number of indicator variables such as transport costs, modal shares etc. The Monte Carlo simulation is based on 168 repetitions.[11]

First we compare the average logistics, transport, order and holding costs (cost/tonnes) in, to and from Sweden with the freight volume in tonnes. We analyze four different type of average costs: total logistics cost per tonne, transport cost per tonne, order cost per tonne and holding cost per tonne. The total logistics cost is the sum of the cost for transportation, ordering and holding. The average cost is calculated as the total cost divided by the total freight volume measured in tonnes per year.

The results are similar to the results in previous section. To analyze the economies of scale we use the same model as in equations (1) and (2). Estimating the scale factor for the average total logistics cost results in a scale factor M equal to -0.53 which implies that if freight demand volume increases with one percent, average logistics cost decreases with 0.53 percent on average. The scale factors for total logistics cost, transport cost, order cost and holding cost are shown in Figure 3 nedan.

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[11] The original analysis was based on 173 repetitions. Five outliers with very large transport volumes were removed. Each run took about 3-4 hours on a server and resulted in around 1.5 GB data which made it difficult to make more runs.
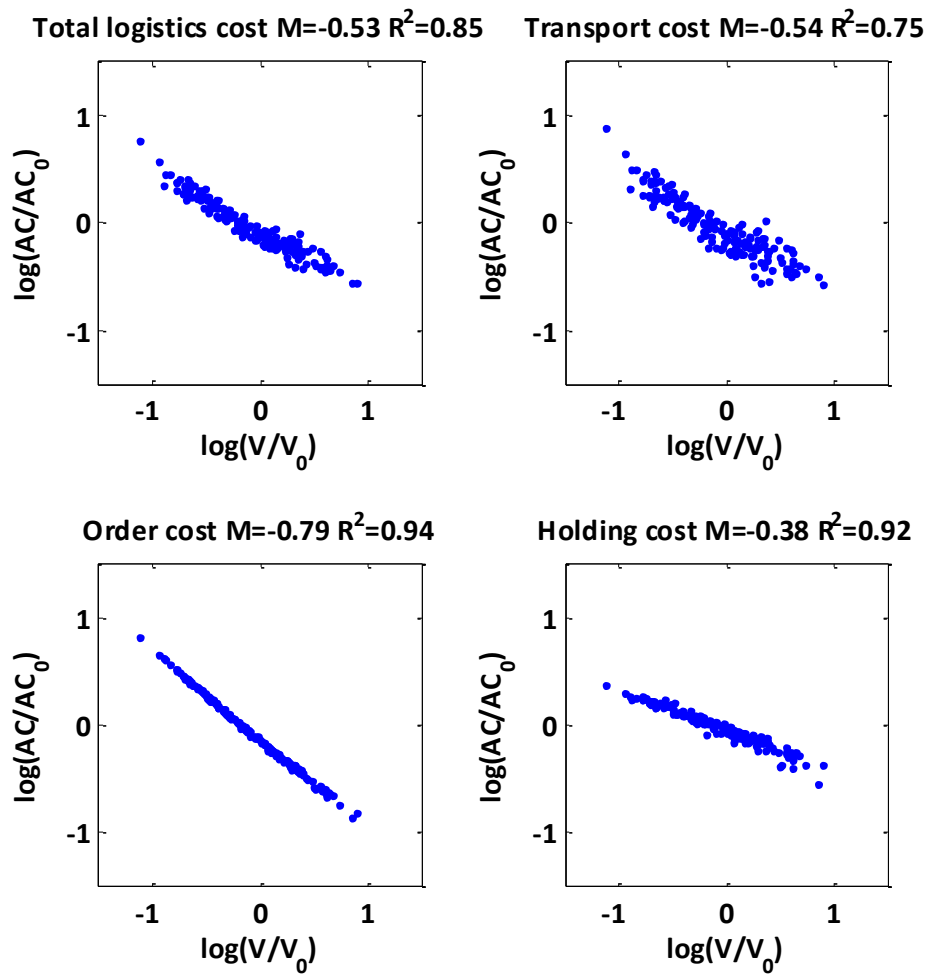
Figure 3: Estimated scale factors for the average logistics cost per tonne, average transport cost per tonne, average order cost per tonne and average holding cost per tonne as a function of total transport demand volume.

The figure shows that the order cost has the most negative scale factor, -0.79, that is, if the total transport volume increases with one percent, the average order cost decreases 0.79 percent on average. With larger transport volumes, the model will suggest larger shipment sizes, hence reducing the average order cost. The relatively large coefficient of determination (R^2=0.94) indicates that this effect is relatively robust. Compared to the average order cost, the scale factor for the average transport cost and average holding cost is less negative, -0.54 and -0.38 respectively. The estimated scale factors in this experiment are hence close to the estimated scale factors in previous experiment. The results are similar to the results in the second experiment with a uniform scale of the total transport demand where the scale factor was found to be -0.47 compared to -0.53 in this experiment.

We also study the effect on modal split as a function of the total freight volume. To do this, we analyze the modal shares for road, rail and sea in Sweden as a function of the total freight volume. The modal shares is shown in Figure 4.
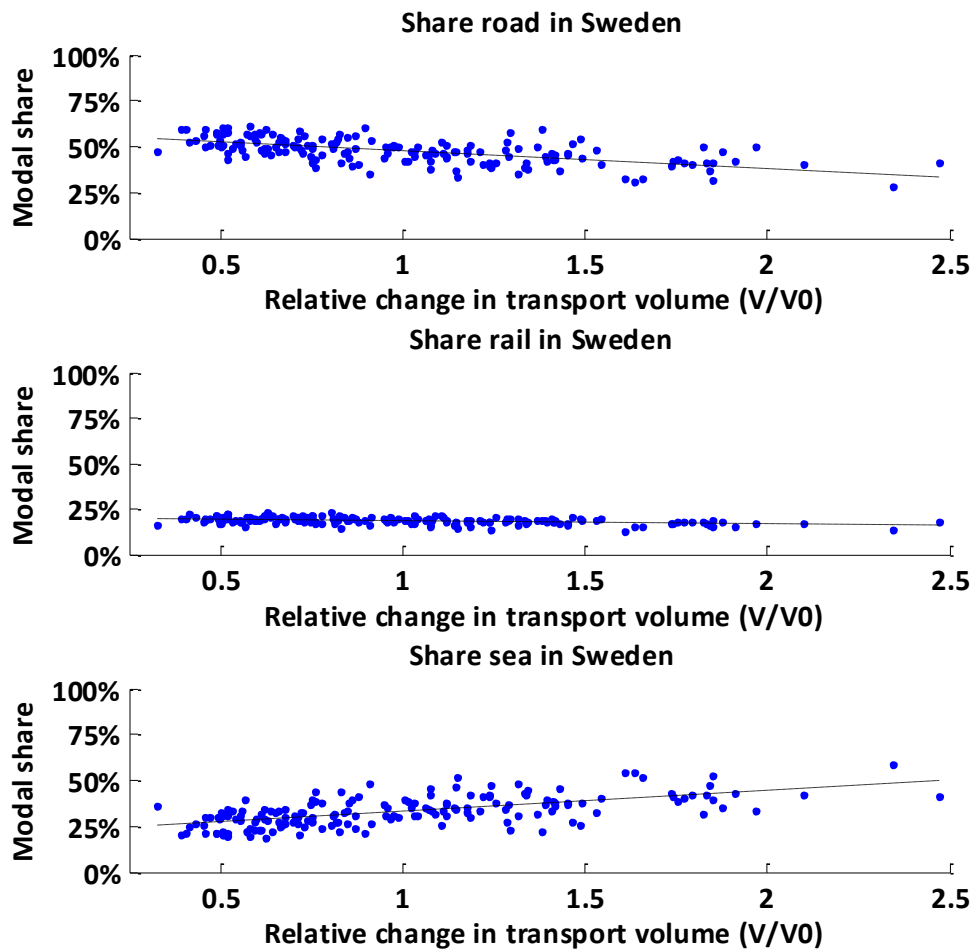
Figure 4: Share of road, rail and sea transport in Sweden as a function of the total freight volume in tonnes per year.

As expected, tonne-kilometers increases with the total transport volume for all modes of transport. The effect is however larger for sea transport than for road and rail transport which makes the share of sea transport to increase at the expense of mainly road transport. This indicates that sea transport has a comparative advantage in utilizing economies of scale when total transport volume increases, which is in line with expectations. This confirms that the logistics model is able to find new transport solutions or other measures to benefit from economies of scale in sea transportation. The data also reveals a positive correlation between total transport volume and load factor for heavy road vehicles, trains and large sea vessels.

3.5   Removal of small flows from the PC-matrices

In the last test, the effect of removing very small flows from the PC-matrices is studied, both in terms of tonne-kilometers and in run time. A constant threshold value for the flows is used; every matrix element less than the threshold value is truncated to zero.
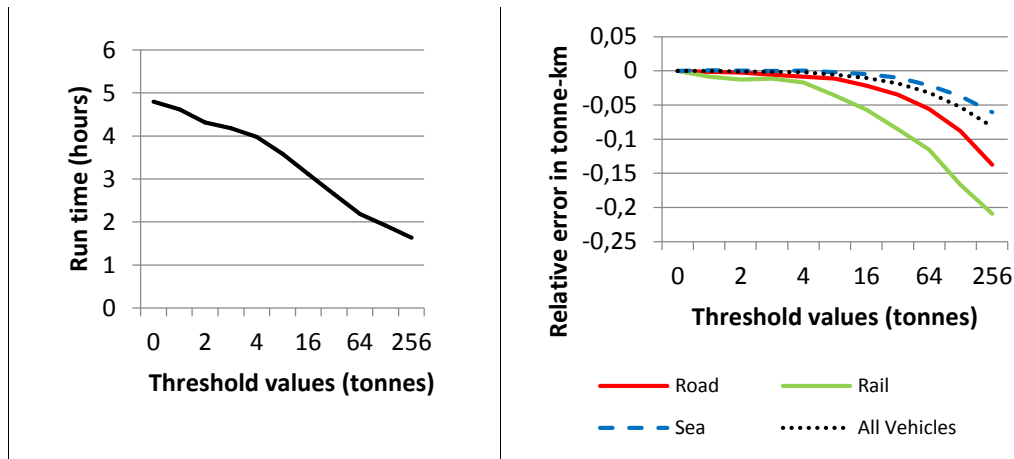
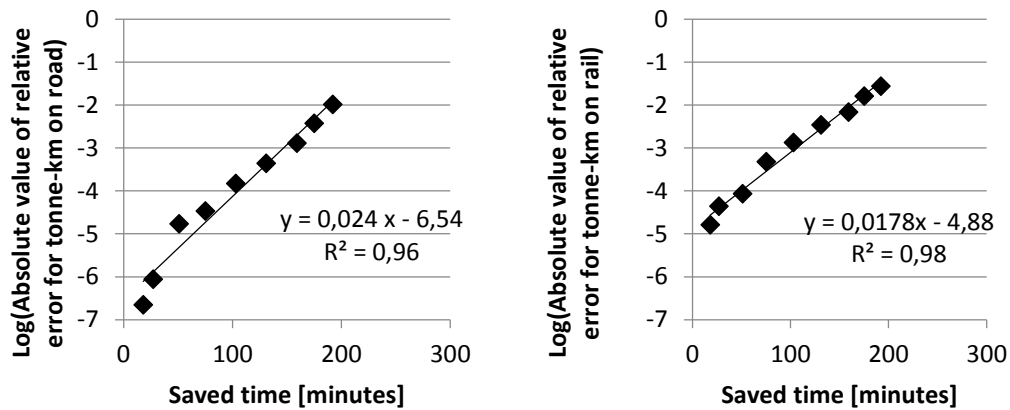*Figure 5: The effect of removing small PC flows on running time and relative error in tonne-kilometer.*

Figure 6: The relationship between running time and precision for road and rail transport.

In Figure 5 the runtime for a complete Samgods scenario computation on the base scenario for 2006 is shown, with varying threshold values. The run time varies almost linearly with the logarithm of the threshold values. There is also an almost linear relationship between the relative error in tonne-kilometer[12] and the threshold values as shown in Figure 6. Consequently, there is an almost linear relationship between run time (and saved time) and the logarithm of the relative error in tonne-kilometers. These empirical findings could be useful for choosing appropriate threshold values for the PC-matrices. The results indicate that it is possible to save a fair amount of run time without effecting aggregate results too much. However, effects on more detailed outputs should be studied before drawing conclusion on an appropriate threshold value.

4   DISCUSSION

The general finding from the experiments with completely independent and completely dependent perturbations is that if the geographical resolution of the PC-matrices is high and the errors of individual PC-relations are independent of each other, then the impacts on the outcomes of the logistics model at an

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[12] Relative error compared to the tonne-kilometer with the original PC-matrices.

aggregate level are very small; increases and decreases in terms of mode shares and costs at this level cancel each other out on average. The outcome becomes a very different one if the geographical resolution of the PC-matrices is low, the perturbation is done at an aggregate level (so that each of the perturbed regions contain many PC-relations that will have perfectly correlated changes) or if there exists a strong correlation between production and/or consumption zones (so that many PC-relations will experience changes in the same direction and by a comparable growth rate).

This paper is, to our knowledge, the first attempt to analyze the sensitivity and economies of scale of a large scale national freight model using Monte Carlo simulation. The results demonstrate that the Swedish national freight model are able to find new logistics solutions and shift freight volumes from land based transports to sea transport when the total volume increases.

An increase in total transport volume by 1 percent will on average reduce logistics cost per tonne by about 0.5 percent. The average order cost has the strongest scale factor (-0.8) which implies that there is a potential for the transport owners in the model to benefit from larger transport volumes by consolidating shipments (i.e. by using larger shipments and lower shipment frequencies). There is also a positive correlation in the simulation between total transport volume and load factor for heavy lorries, trains and large ships. Without empirical data and further analysis, it is however difficult to assess whether the strength of this effect also corresponds to reality.

According to the large importance of the input data in the deterministic Samgods logistics model, in-depth studies for the demand of different commodities and the application of more indicators (such as load factors, throughput in major ports, truck and freight train flows on main infrastructure links) are desirable. Time series data can also be used to validate how shipment sizes and consolidation vary with changes in the total transport volume. Another question is whether errors in the PC-matrices lead to larger or smaller errors in the model outputs (e.g. tonne- kilometer by mode). It would also be interesting to study the impact of revised logistics costs on the outcomes of the Samgods logistics model. Results from this analysis can be used in the production of future demand matrices and transport forecasts (including sensitivity analyses).

A challenge in the project was the relatively long run times for each simulation. With more features added to the model, for instance by including rail capacity constraints, the run times will increase. The fourth experiment indicates that the run times can be reduced by removing small transport flows without affecting aggregate results too much. To verify this result more research on the tradeoff between accuracy and running time is needed.

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