Estimating the marginal cost of different vehicle types on rail infrastructure

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Abstract

In this paper we combine engineering and economic methods to estimate the relative cost of damage mechanisms on the Swedish rail infrastructure. The former method is good at predicting damage from traffic, while the latter is suitable for establishing a relationship between damage and cost. We exploit the best features of both methods in a two-stage approach and demonstrate its applicability for rail infrastructure charging. Our estimations are based on 143 track sections comprising about 11 000 km of tracks. We demonstrate how the estimated relative costs of damage mechanisms can be used in order to calculate the marginal wear and tear cost of different vehicle types. The results are relevant for infrastructure managers in Europe who desire to differentiate their track access charges such that each vehicle pays its short run-marginal wear and tear cost, which can create a more efficient use of the rail infrastructure.

Keywords: marginal cost, rail infrastructure, maintenance, access charging, track damage, econometric methods, and engineering simulation

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Abstract

In this paper we combine engineering and economic methods to estimate the relative cost of damage mechanisms on the Swedish rail infrastructure. The former method is good at predicting damage from traffic, while the latter is suitable for establishing a relationship between damage and cost. We exploit the best features of both methods in a two-stage approach and demonstrate its applicability for rail infrastructure charging. Our estimations are based on 143 track sections comprising about 11 000 km of tracks. We demonstrate how the estimated relative costs of damage mechanisms can be used in order to calculate the marginal wear and tear cost of different vehicle types. The results are relevant for infrastructure managers in Europe who desire to differentiate their track access charges such that each vehicle pays its short run-marginal wear and tear cost, which can create a more efficient use of the rail infrastructure.

1.0 Introduction

Operating a train service generates costs for the management of the rail infrastructure. Research on these costs became relevant for European policy after the vertical separation between infrastructure management and train operations in the 1990s, requiring track access charges to be set. To create an efficient use of the infrastructure, each vehicle should at least pay its short-run marginal cost, which is a requirement supported by EU legislation (see European Commission Directive 2012/34/EC).

One component of the costs incurred by a train service is the wear and tear of the rail infrastructure. The vertical force on the tracks created by the weight of the train is an important factor for this damage, and ton-km has been the most widely applied charging unit in Europe. However, the damage per ton-km can vary depending on the vehicle type used, where the number of axles and bogie type are important characteristics in this respect. Differentiating the track access charge with respect to the variations in damage and cost incurred by different vehicle types creates stronger incentives for running more "track friendly" vehicles, and would create an even more efficient use of the infrastructure compared to a ton-km charge. Britain and Switzerland are examples of European countries that have chosen to differentiate their track access charges by vehicle type and ton-km. This type of charge requires an estimation of the marginal cost of different vehicle types running on the rail infrastructure, which is the purpose of this paper.

Different approaches have been used in the literature to determine the marginal wear and tear cost. The top-down approach tries to establish a direct relationship between costs and traffic using econometric techniques (see for example Link et al. 2008 and Wheat et al. 2009), while the bottom-up approach uses engineering models to estimate the damage caused by traffic, damages that are then linked to maintenance and renewal activities and their respective costs (see for example Booz Allen Hamilton 2005 and Öberg et al. 2007). A combination of these approaches has been proposed by Smith et al. (2014): a two-stage approach in which simulation methods (engineering models) are used in the first stage to estimate the track damage caused by the rail vehicles running on the tracks. The relationship between damage and costs are then established using econometric methods in the second stage.

The reason for combining the econometric and engineering approaches in this type of exercise is that they can complement each other. The strength of the former approach is that it uses actual costs and can put few restrictions on the elasticities of production. However, it has difficulties in picking up the complexity of the relationship between different vehicle types and costs. The engineering approach is on the other hand good at estimating the relative damage caused by different vehicles, but has difficulties in linking the damages (caused by traffic) to actual costs.

Smith et al. (2014) applied their approach on Swedish data comprising 45 track sections, in order to demonstrate the feasibility of the method. In this paper, we apply the same two-stage approach with an aim to increase the precision of the marginal cost estimates. The contribution of this paper is therefore to test if the two-stage approach is applicable for charging purposes; can it be a viable approach for infrastructure managers that wish to differentiate their track access charges by vehicle type? To do so, we use a significantly larger dataset comprising 143 track sections in Sweden. Moreover, the simulation stage of the approach is refined, with significantly more detailed vehicle models in the simulation stage compared to the previous study. Hence, we make fewer assumptions on the damage caused by certain vehicles on the track. An extra damage mechanism is also included in our study.

The outline of the paper is as follows. The methodology is described in section 2. Sections 3 and 4 present the estimation stages in our approach in more detail. A description of the data is given in section 5. The estimation results are presented in section 6 together with a demonstration of the marginal cost calculations. Section 7 concludes.

2.0 Methodology

The econometric (top-down) approach and the engineering (bottom-up) approach are the two main methods to determine how wear and tear costs of the rail infrastructure vary with traffic. The former has become the most widely used approach, and its results are applied in many European countries. Munduch et al. (2002) and Johansson and Nilsson (2004) are early examples of studies that use a (translog) cost function in order to derive a cost elasticity with respect to traffic using econometric techniques; an elasticity showing how a proportionate increase in traffic affects costs proportionately (this cost elasticity is multiplied with the average cost in order to get a marginal cost estimate). A set of control variables are included to account for heterogeneity in the production environment - that is, to isolate the effect traffic has on costs. This method has however not been successful in isolating the effect different vehicle types has on wear and tear costs, and the traffic measure used in econometric studies is generally gross tons.

Instead of trying to establish a direct relationship between traffic and costs, the bottom-up approach uses engineering models to determine the damage caused by traffic. The damage is often categorized as rolling contact fatigue, track settlement, or wear of the rail. The method is able to provide estimates of how much of each type of damage that different vehicles has caused, with the possibility to account for current infrastructure characteristics such as track geometry and rail profiles. These damages are then linked to costs in order to produce a marginal cost. This can be done with information on the volume of activities made to rectify the damage caused by traffic and the unit cost of those activities. Costs can then be allocated to the different vehicle types. This approach has been used in Britain (see for example Booz Allen Hamilton 2005 and ORR 2013). However, a critical point in the approach concerns the relative cost of the different types of damages. For example, Öberg et al. (2007) use an engineering approach to produce estimates of the amount and type of damage caused by different vehicles on the Swedish railway network. The damage types are then assigned shares of maintenance and renewal costs based on advice from experts within the Swedish Rail Administration[1]. The experts' advice may, or may not be, close to the actual cost shares. In general, the link from damages to costs needs to account for external factors such as a heterogeneous production environment, which can vary in aspects that are difficult to capture without a statistical (econometric) approach. Examples are rail age (proxy for accumulated use) and track quality.

The approached proposed by Smith et al. (2014) is to use an econometric model to estimate the share of costs that can be attributed to the different damages - that is, the relative cost of the damage types. The same approach is used in this paper.

More specifically, the estimation approach (depicted in Figure 1 below) consists of two stages. Similar to the bottom-up approach, we perform simulations based on engineering models in the first stage. We use traffic data together with infrastructure characteristics in order to simulate four different damage mechanisms: track settlement, wear of rails, rolling contact fatigue (RCF), and track component fatigue. Hence, we include an extra damage mechanism (track component fatigue) compared to the study by Smith et al. (2014). This damage mechanism may eventually require replacements of components and can be important to consider given that minor replacements are defined as maintenance.

The output from the first stage is measures of the different damage types per ton-km for each vehicle type. Apart from differences in traffic between track sections on the rail network, these damage measures can also vary for each section due to the different characteristics of the sections such as track geometry and curvature. The measures are then scaled up based on the traffic volume of each vehicle type on the different track sections. In that way, we produce measures on the total track component fatigue, track settlement, RCF

[1] This organization merged with the Swedish Road Administration in 2010, forming the Swedish Transport Administration.

and wear of rails, that traffic has caused on a section. We use these damage measures in the second stage, in which a statistical model is formulated where maintenance cost is a function of the damage mechanisms and other cost drivers. Cost elasticities are derived from the statistical model, giving us the relative cost of the damage types. Based on the information from the simulation, we can estimate the marginal cost of the vehicle types.

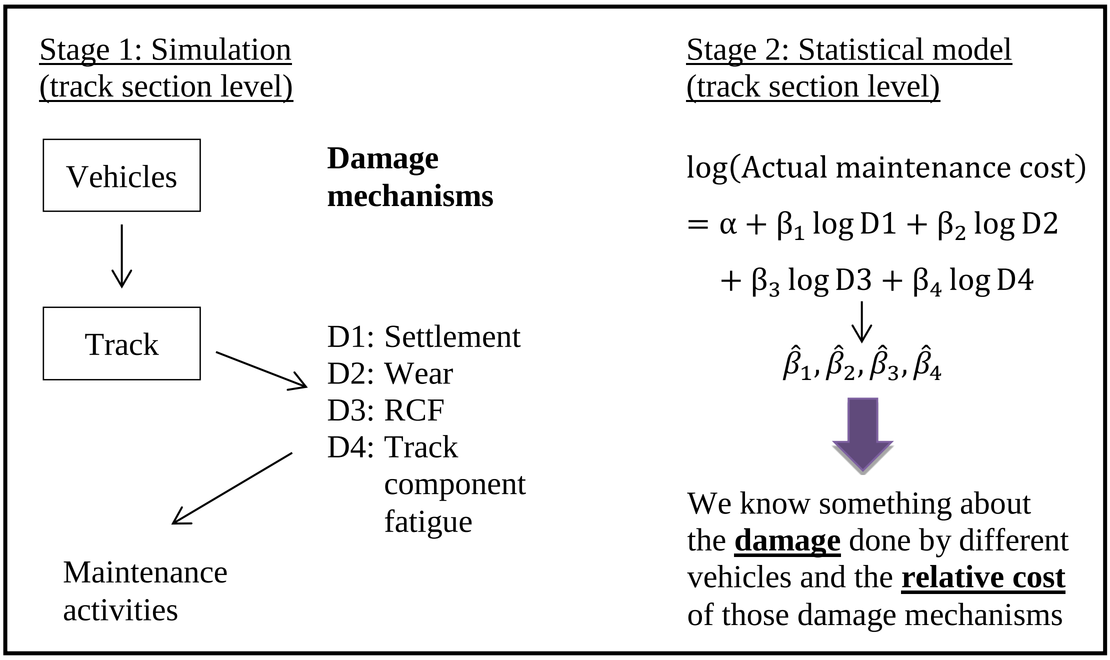

Figure 1: Overview of the methodology (revised figure from Smith et al. 2014)

A detailed description of the simulations and the econometric model we estimate is provided in sections 3 and 4 respectively.

3.0 First stage: simulations

Calculating the amount of track damage is a complex matter. First, it is a function of track quality itself - that is, whether

- the track is newly built or recently repaired,
- the sleepers are wooden or concrete, or if it is a slab track,

- the environment is humid or dry,
- wheel and rail profiles are well matched or not.

We account for the track quality in the simulations by using measurements on the track geometry, which will differ depending on the age of the track or if it has recently been repaired. However, we only consider concrete sleeper tracks and a constant environment (which creates a constant friction level) in the simulations. Moreover, we do not account for the actual matching of the wheel and rail profiles as it is beyond the scope of this paper. This means, only new unworn wheel and rail profiles are used.

The vehicles operating on the track are also of great importance for the damage incurred. Some of these determining factors - that are taken into account in our simulations - are

- bogie design,
- wagon structure,
- axle load and
- vehicle speed.

The track damages investigated in this study are categorised into four different types. These are:

- track component fatigue,[2]
- track settlement,
- rolling contact fatigue (RCF) and
- wear of rails.

[2] Additional measure not included in original work by Smith et al. (2016).

We quantify and calculate the amount of track damage for each of the damage types listed above, using dynamic simulation and the damage prediction models available in the literature. The simulations are performed on 143 track sections in Sweden, which in total comprise about 11 000 km of tracks. Traffic data from 2014 are used in order to identify the vehicle types running on each track section. This includes information on the number of vehicles operating on each track section as well as the vehicle types and their ton-km values. The dynamic simulations and the modelling issues are described in more detail in the following section.

3.1 Dynamic simulations

Computer based vehicle dynamics calculations using multibody simulation software have been widely used by companies and researchers for many years. These simulations are mainly used to predict the dynamic behaviour of the vehicles for different track conditions and operating conditions, thus making sure that the requirements on derailment, ride quality, track forces, RCF and wear will be met. It is also a very powerful tool to reduce the number of expensive field measurements. The modelling issues in these dynamic simulations are divided into three parts: track models, vehicle models and the wheel-rail contact.

We use a track model representing concrete sleeper tracks in the simulations, which is the sleeper type used on most of the tracks in Sweden (see Chaar and Berg (2006) for more information on track flexibility characteristics and its validation). The model of the track comprises of ground, ballast, rails and stiffness between these bodies as shown in Figure 2.

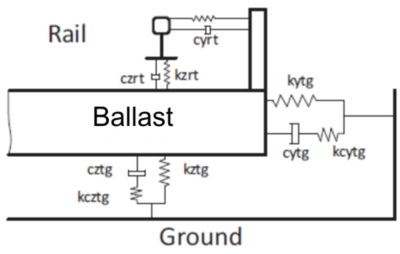

Figure 2: **Example of model for track flexibility**

In principle, vehicle modelling starts with defining the rigid or flexible bodies connected by springs, dampers and links. The bodies include car body (passenger wagons or freight baskets), bogie parts (frames, bolster beams, and possible steering links) and axles (axle box and wheels). A four axle bogie vehicle may be modelled as seven rigid bodies which are shown in Figure 3. These rigid bodies may have all six degrees of freedom unless they are constrained. These six motions are vertical, lateral, longitudinal, pitch, yaw and roll.

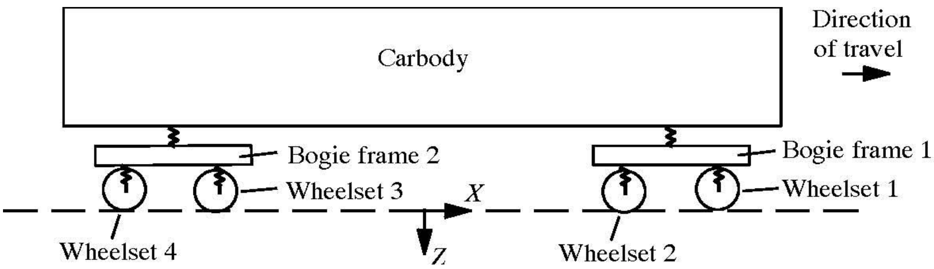

Figure 3: **schematic model of a four axle bogie vehicle**

For each rigid body mass we have to know the moments of inertia, nominal positions of the centre of gravities, and locations of the coupling elements. The other important part is modelling the suspension elements. These elements are mainly springs, dampers and frictional contacts. Some of these elements are heavily non-linear and their behaviour depends on the applied loads, forces and displacements. A suspension element could be a coil or rubber spring, air spring, leaf spring, hydraulic damper, and metal wedge. For each type of element mentioned, there is a mathematical model which represents its dynamic behaviour.

The vehicle models we choose depend on the traffic running on the 143 track sections in this study. According to the traffic data, there were 111 rail vehicles in total operating on these sections in 2014. It is not possible to model each of these vehicles separately. Thus, the vehicles are categorized based on the type of the running gear, vehicle category (freight/passenger), axle load and maximum speed. The chosen categories are presented in Table 1. Moreover, due to time restrictions, we only run simulations for vehicles that comprise more than 9 per cent of a track section's total ton-km. The vehicles that are left out are assigned the damage values from simulated vehicles with the most similar characteristics with respect to damage.

Table 1: **Vehicle model categories with their maximum speed**

Categories	**Max. speed km/h**
Motor coach 4x16 t*	200
Passenger car 4x14 t	160
Motor coach 4x16 t**	200
Motor coach 4x12 t*	140
Motor coach 4x21 t, high centre of gravity**	200
Motor coach with Jacob bogie 3x16.5 t**	160
Motor coach with Jacob bogie 3x12.5 t*	200
Freight loco 6x20 t	120
Freight loco 4x20 t	120
Freight loco 6x30 t	70
Passenger loco 4x19 t	140
Passenger loco 4x19 t	175
Freight wagon (2x22 t or 2x6.5 t)	100
Three-piece bogie 4x30 t	60 (laden)
Three-piece bogie 4x6.5 t	60 (tare)
Y25 bogie 4x22 t	100

* Flexible wheelset guidance, ** Stiff wheelset guidance

All the mentioned models are carefully designed and the results of the calculations are validated against the field measurements for certain types of the vehicles. To design and run the simulation models the Swedish multibody simulation software GENSYS (2015) is used.

Lastly, we need to model the wheel and rail contact, which is the tiny contact area between the wheels and the rails that is subjected to very high stresses. The way to calculate these stresses is crucial for prediction of the dynamic interaction between the vehicle and the track. A wheel-rail contact model consists principally of a wheel-rail geometry module, a creep/spin calculation procedure and a creep force generator. The theories are described for example in Andersson et al. (2015). In this study, the Hertzian solution and Kalkers FASTSIM method is used for the normal and tangential contact problem respectively.

3.2 Simulation inputs

Inputs needed for the simulation are ideal track geometry and track irregularities, vehicle speeds, wheel and rail profiles, and axle loads. Data on track geometry has been provided by the Swedish Transport Administration (Trafikverket), and originates from track measurements in 2014. The geometry includes the longitudinal position on the line, track super-elevation, track lift and track curvature. The irregularities include lateral, vertical, cant and gauge irregularities.

We set the vehicle speed as a function of cant deficiency in a way that maximum allowed cant deficiency can be reached, where the maximum lateral acceleration will be limited according to Banverket (1996). There are three categories defined based on the vehicles running gears,

- category A; conventional vehicle with older running gear $a_{y,lim} = 0.65\ {m}/{s^2}$,
- category B; vehicles with improved running gear $a_{y,lim} = 0.98\ {m}/{s^2}$,

- category C; X2000 and other high speed trains $a_{y,lim} = 1.60\ {}^{m}/_{s^2}$.

The maximum vehicle speed is limited with the permissible speed on each line.

As wheel and rail material will be worn gradually, their profiles' shape also changes. Therefore it is almost impossible to use all the actual wheel-rail profiles in operation. However, as the majority of rails in Sweden have UIC 60 and the wheels have S1002 profiles, we use these profiles in the simulations. The rail inclination in Sweden is 1:30.

Apart from the Iron-Ore locomotives and wagons, which run at 30t axle load, the rest of the freight wagons are simulated with 22.5t axle load when they are fully loaded. The weight of the empty freight wagons is calculated based on their basket, bogie and other parts' design. For passenger trains there is no generally accepted standard for calculation of the passenger loads. However, the weight of a passenger including hand luggage can be estimated to 80 kg. According to European standard EN 12663 (CEN 2010), the number of passengers in a coach is equal to the number of seats, which is the standard we use.

3.3 Track damage

We calculate four types of track damages for each vehicle on each track section: track settlement, track component fatigue, wear of rails and rolling contact fatigue (RCF).

Track settlement has a major influence on maintenance cost and is usually caused by high wheel-rail forces from passing vehicles. This type of damage depends strongly on the amount of track irregularities. Thus, axle load, unsprung mass and speed, track construction, track quality and track condition are the most important factors determining the magnitude of the damage. To calculate the settlement damage, various empirical models have been used. However, in most of them, the vertical wheel-rail force raised to a power is used as a damage indicator. In the present study the adapted TUM (Technical University of Munich) settlement calculation model is used:

$$Settlement = A \cdot Q^{1.21} \log \mathrm{N} \tag{1}$$

where,

N = number of axles passes

Q = Vertical force at the wheelset

A = constant; (A=1 in the current work)

Internal fatigue damage due to repeated loading is a function of both vertical and lateral track forces. The components affected by the repeated loading are rails, rail pads, rail fasteners, and sleepers. The calculation method is developed by UIC/ORE (1987) based on extensive tests and it is complemented by Öberg et al. (2007) with a lateral force component – that is, the resulting force on either rail.

$$Track\ component\ fatigue = \sum_{i=1}^{n_v} \sqrt{{Q_{tot\_i}}^2 + {Y_{qst\_i}}^2}^{\,3} \tag{2}$$

where,

n_v= number of axles

Q_{tot_i} = total vertical force including quasistatic and dynamic forces

Y_{qst_i} = quasistatic lateral force

Wear of rail and wheel is a function of material properties (steel grade), contact pressure (axle load, wheel-rail profile), sliding velocity (creepage and spin), weather condition (sun and rain) and lubrication (track side or vehicle based). In this study the friction level is assumed to be 0.45 for all the simulations unless the locomotives are equipped with vehicle based lubrication systems - that is, the Iron-Ore loco. To predict the wear on rails, several prediction models are proposed in literature (see Enblom 2004). One of the most widely used and simple ways to predict the amount of wear is to calculate the dissipated energy in the wheel-rail contact patch. This is based on an assumption that there is a linear relationship between wear and energy dissipation. Energy dissipation per meter running distance can be calculated as:

$$\boldsymbol{\bar{E}} = \boldsymbol{F_x \nu_x} + \boldsymbol{F_y \nu_y} + \boldsymbol{M\varphi} \qquad (3)$$

where,

$\boldsymbol{F_x}$ and $\boldsymbol{F_y}$ are longitudinal and lateral creep forces,

$\boldsymbol{\nu_x}$ and $\boldsymbol{\nu_y}$ are longitudinal and lateral creepages,

M is the moment and φ is the spin in the contact patch.

In the present study it is assumed that if the wear values are below 160 J/m, then the wear regime is considered to be mild wear and the value of wear damage is neglected (Smith et al. 2014).

To calculate surface initiated cracks due to RCF, again the energy dissipation based theory is used (see Figure 4). Here, first the energy dissipation is calculated and then the RCF index is picked accordingly.

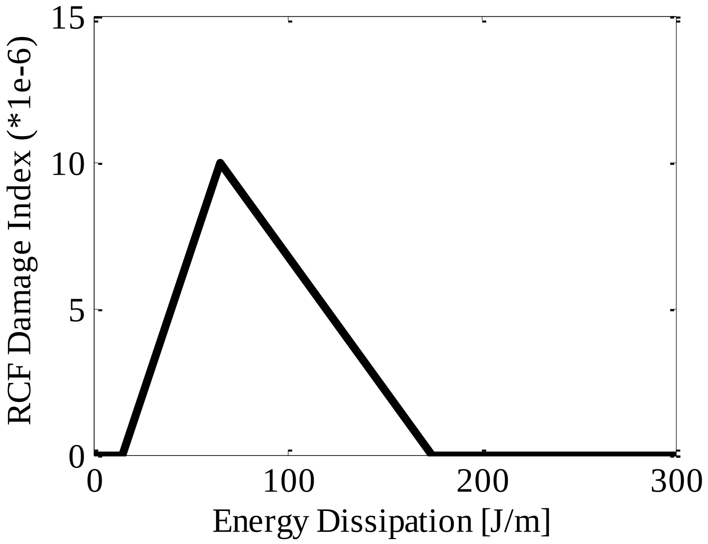

Figure 4: **Rail RCF damage function (Burstow 2004)**

3.4 Time domain analysis

All equations of motions have to be integrated numerically in each time step. The results of each time step are the inputs for the next one. This is called initial value numerical calculations. In this study 1ms is used for the time steps. Depending on the track section length, the vehicle model, vehicle speed and the track quality, each simulation corresponding to 1 km of the line takes around 1 to 10 minutes not including the time needed for post-processing of the results. Therefore it is basically impossible to perform the simulations for all vehicles on the entire length of all track sections. Instead, we use the load collective method, which is also used in publications such as Enblom (2004) and Dirk and Enblom (2011).

More specifically, this method implies that we create 10 different subsection categories as a function of the track curvature. These subsection categories are track pieces with radii 0-400m, 400-600m, 600-800m, 800-1000m, 1000-1500m, 1500-2000m, 2000-3000m, 3000-5000m, 5000-10000m and above 10000m. Hence, a track section has many track pieces in each subsection category. Considering that we cannot run simulations on the entire track, we choose one track piece in each subsection category (measured by the track geometry car). Specifically, we choose the piece with a track length that is closest to the mean length of all the track pieces in its subsection category. The simulated damage on each piece is then scaled up with respect to the total track length of the subsection category the piece belongs to.

3.5 Simulation results

All four track damage values are calculated for all the subsection categories on each track section and for every vehicle operating on that specific section. Maximum values are considered for all types of damages. The values are then summed for all axles and scaled based on the contribution of the subsection to the entire track section and normalised by the ton-km values obtained from the traffic data.

To show the evaluation process, we present the calculation for a part of track section 217 (see Figure 5). As mentioned earlier, the line is divided into 10 different subsections, depending on the curve radii, and the length of each subsection is presented in Table 2.

Table 2: **Subsection lengths of section 217 (based on route length)**

Subsection	**0-400m**	**400-600m**	**600-800m**	**800-1000m**	**1000-1500m**	**1500-2000m**	**2000-3000m**	**3000-5000m**	**5000-10000m**	**Straight**
Total length (m)	0	3543	1584	4368	4139	845	639	565	339	46 627

The traffic data shows that there are eight vehicle categories operating on this line. The corresponding ton-km of each vehicle type is presented in Table 3.

Table 3: **Vehicle types & the corresponding ton-km values for section 217**

Vehicle types	Million ton-km
Motor coach 4x12t, V_{max} 140 km/h*	0.06
Passenger car 4x14 t, V_{max} 160 km/h	17.89
Motor coach 4x16 t, V_{max} 200 km/h**	56.97
Freight loco 6x20 t, V_{max} 120 km/h	1.63
Freight loco 4x20 t, V_{max} 120 km/h	0.54
Passenger loco 4x19 t, V_{max} 140 km/h	61.45
Passenger loco 4x19 t, V_{max} 175 km/h	19.88
Freight wagon 2x22 t, V_{max} 100 km/h	38.72
Freight wagon 2x6.5 t, V_{max} 100 km/h	12.51
Y25 bogie 4x22 t, V_{max} 100 km/h	659.03
"Unkown"	1.39

*Flexible wheelset guidance, ** Stiff wheelset guidance

Figure 5: location of section 217

The calculated damages for a "freight loco 4x20 t, V_{max} 120 km/h" running on six segments that constitutes one track piece is presented in Table 4. The sum of the maximum wear number for all the axles of the first and the second bogie of this vehicle type between 83910m to 84510m on section 217 is 1423 J/m. This particular track piece belongs to the curve interval 600-800m, and is scaled accordingly. The total wear on these tracks is: (1423/600)*1584 = 3575 J/m.

The same type of calculations are performed for the rest of the curve intervals, including straight lines, in order to produce values of the total wear, RCF, settlement and track component fatigue incurred by a "freight loco 4x20 t, Vmax 120 km/h"-vehicle. Using the weight of this vehicle and the route length of the track section, we calculate its damage values per ton-km. With information on this vehicle type's total ton-km on track section 217, we can scale up the total damage caused by this vehicle on this section. The same type of simulations and calculations are made for rest of the vehicles running on this section in order to produce measures of total wear, RCF, track settlement, and track component.

Table 4: **Results for curve interval 600-800m on section 217 for a freight loco 4x20 t, V_{max} 120 km/h**

Longitudinal position Start_Stop (m)	Wear	RCF	Settlement	Component
83910_84010	134	1.30	5 799 725	3.60E+16
84010_84110	197	2.24	5 740 527	3.52E+16
84110_84210	326	2.96	5 774 048	3.58E+16
84210_84310	322	2.83	5 678 586	3.44E+16
84310_84410	276	2.77	5 679 442	3.43E+16
84410_84510	168	1.84	5 791 845	3.59E+16
Sum	**1423**	**13.94**	**34 464 175**	**2.12E+17**

4.0 Second stage: Econometric model

With estimates on the damage caused by traffic, we can derive cost elasticities for the damage types using econometric methods. To do so, we need to control for other factors that may influence maintenance costs, such as the average rail age on a section. More specifically, we formulate costs as a function of a set of variables, where the damage types are the variables of main interest

$$C_i = f(D_{1i}, D_{2i}, D_{3i}, D_{4i}, \boldsymbol{X}_i), \tag{4}$$

where C_i is maintenance costs on $i = 1, 2, \ldots, N$ track sections. $D_{1i}, D_{2i}, D_{3i},$ and D_{4i} are the damage types track settlement, wear of rails, RCF and track component fatigue. $\boldsymbol{X}_i$ is a vector of infrastructure characteristics such as track length and the average age of rails.

As described previously, the damage measures are based on the total ton-km on each section, which in turn depend on the length of each section. Therefore, to separate track length effects from damage effects, we use damage density variables ($\frac{D_{1i}}{Track-km_i}$, $\frac{D_{2i}}{Track-km_i}$ etc.) along with the track length variable in the model estimations.

In our estimation approach, we start with the translog model proposed by Christensen et al. (1971), which is a second order approximation of a cost (production) function (see for example Christensen and Greene 1976 for an application to cost functions). Both the dependent variable (costs) and the independent variables (damages and infrastructure characteristics) are subject to a logarithmic transformation in this model, which can reduce skewness and heteroscedasticity (problems that may invalidate the statistical inference if not treated correctly). Specifically, we consider A damage types, K network characteristics and M dummy variables, and express the model as

$$lnC_i = \alpha + \sum_{a=1}^{A} \beta_a lnD_{ai} + \frac{1}{2}\sum_{a=1}^{A}\sum_{b=a}^{A} \beta_{ab} lnD_{ai}\, lnD_{bi} + \sum_{k=1}^{K} \beta_k lnX_{ki} + \frac{1}{2}\sum_{k=1}^{K}\sum_{l=a}^{K} \beta_{kl} lnX_{ki}\, lnX_{li} + \sum_{a=1}^{A}\sum_{k=1}^{K} \beta_{ak} lnD_{ai} lnX_{ki} + \sum_{m=1}^{M} \beta_m Z_{mi} + v_i \tag{5}$$

where α is a scalar, v_i is white noise and $\boldsymbol{\beta}$ is a vector of parameters to be estimated. The simpler (and more restrictive) Cobb-Douglas model is nested in the translog model. We check the Cobb-Douglas constraint $\beta_{ab} = \beta_{kl} = \beta_{ak} = 0$ using an F-test.

5.0 Data

In total, there were 244 track sections in 2014 administered by the Swedish Transport Administration and their five regional units: Region North, West, East, South and Central. However, limited access to up-to-date track geometry data constrains us to analyze a somewhat smaller part of the Swedish railway network. One may therefore suspect the presence of a selection bias in our data. However, the 143 sections in our data set cover

11 000 track-km out of the 14 100 track-km administered by the Swedish Transport Administration. Hence, the tracks in our data comprise a cross-section of the Swedish rail network with sections from north to south and with large variations in traffic and costs (see Table 5). Still, we are able to compare the 143 track sections with 169 track sections for which we have information on costs, network characteristics and traffic data. Descriptive statistics of the data are provided in Table 5 (143 sections) and in Table 14 in appendix (169 sections). Estimating a translog cost model generates cost elasticities with respect to ton density at 0.2024 (robust std. error is 0.0479) and 0.2258 (robust std. error is 0.0498), using 143 and 169 track sections, respectively.[3] We therefore consider a (possible) selection bias to be a minor issue in our sample.

The costs for rectifying track damage are defined as either maintenance or renewal costs. The former are costs for activities conducted in order to preserve the railway's assets, while the latter are costs for major replacements (minor replacements are defined as maintenance). Given the lumpy nature of renewals, and that we only have access to data for one year (2014), we limit our analysis to maintenance costs only.

Information on the infrastructure characteristics has mainly been collected from the Transport Administration's track information system (BIS), and comprises data on track length, rail age and quality classification (track geometry requirements linked to maximum line speed allowed). As noted in section 3.0, the traffic data contains information on the vehicles (type of wagons, locomotives, multiple unit trains) and their ton-km. The vehicles have been categorized as previously shown in Table 1.

[3] The difference in these estimates is not statistically significant at the 1 per cent level (cf. Cohen et al. 2003, p.46-47).

Table 5: Descriptive statistics, obs. from 143 track sections

	Median	Mean	St. dev.	Min	Max
Maintenance costs, million SEK	14.25	19.66	17.86	0.87	108.67
Wear	2.21E+12	8.22E+14	5.32E+15	8.26E+06	5.52E+16
RCF	5.58E+08	6.55E+11	3.93E+12	4.46E+05	3.37E+13
Settlement	7.46E+14	5.87E+15	2.75E+16	4.61E+11	2.54E+17
Track component fatigue	3.91E+24	1.85E+28	1.44E+29	4.11E+21	1.38E+30
Wear density	1.09E+08	2.43E+08	4.38E+08	2.10E+06	2.75E+09
RCF density	3.15E+05	5.12E+05	7.46E+05	1.02E+04	7.84E+06
Settlement density	2.96E+12	3.66E+12	3.30E+12	4.89E+10	2.45E+13
Track component fatigue density	2.18E+22	3.37E+22	4.87E+22	3.83E+20	4.80E+23
Route length, km	50.17	60.86	40.59	5.97	215.95
Track length, km	63.95	78.79	52.41	7.84	251.39
Average quality class*	2.77	2.74	1.08	1.00	5.02
Average age of rails	21.2	22.4	9.4	4.1	51.3
Million ton density	4.23	7.68	8.24	0.11	45.72
Region West	0	0.20	0.40	0	1
Region North	0	0.13	0.33	0	1
Region Central	0	0.17	0.38	0	1
Region South	0	0.29	0.45	0	1
Region East	0	0.22	0.42	0	1

* Track quality class ranges from 0-5 (from low to high line speed), but 1 has been added to avoid observations with value 0.

6.0 Results

Two models are estimated using ordinary least squares (OLS) and the results are presented in Table 6. Model 1 only includes the damage measures, while Model 2 also includes infrastructure characteristics and dummy variables for the regional units, showing the importance of controlling for the production environment in the estimation. All estimations are carried out with Stata 12 (StataCorp.2011).

However, as a starting point, we examine the correlation coefficients between the different damage mechanisms, which are presented in Table 6. These are all quite high. Track settlement covaries strongly with track component fatigue (the correlation coefficient is 0.95) and with RCF (0.82). The correlation coefficient for wear and track settlement is the lowest

(0.72). We therefore also estimate our models using only these two damage mechanisms (*Model 1c*), as we expect them to capture the effects of RCF and track component fatigue to a large extent.

Table 6: Correlation coefficients

	Wear_den.	RCF_den.	Settl. _den.	Comp._den.
Wear_den.	1.0000			
RCF_den.	0.7228	1.0000		
Settl._den.	0.7155	0.8157	1.0000	
Comp._den.	0.8123	0.7752	0.9471	1.0000

As noted in section 4, we start with a full translog model and test linear restrictions of the parameter estimates using F-tests, which results in the restricted translog models presented in Table 7 and 8.

In Model 1, we note that the estimated cost elasticity with respect to track component fatigue is negative (and statistically significant). This result is counterintuitive; indicating that 10 per cent more track component fatigue will *lower* maintenance costs with about 6 per cent. However, the variance inflation factors (VIFs) for the first order coefficients for settlement and track component fatigue are 0.17 and 0.20, respectively. Also, considering the high correlation coefficients, we drop track component fatigue, which results in *Model 1b*. The first order coefficient for settlement then falls from 0.62 to -0.03 (not significantly different from zero). Dropping RCF due to its high correlation with settlement (0.82), results in Model 1c. The sum of the first order coefficients are rather similar in the models (0.44, 0.33 and 0.40), which indicates that the strong correlation between the damages mechanisms affects the individual parameter estimates significantly.

Table 7: Estimation results, Model 1

	Model 1a		*Model 1b*		*Model 1c*	
	Coef.	Rob. Std. Err.	Coef.	Rob. Std. Err.	Coef.	Rob. Std. Err.
Cons.	16.5063***	0.1124	16.3996***	0.0763	16.5134***	0.0826
Wear_den.	0.3616**	0.1495	0.0137	0.0974	0.2845**	0.1151
Wear_den.^2	-0.4863**	0.2443	-	-	-0.3613***	0.1183
RCF_den.	0.0885	0.1520	0.3544***	0.1195	-	-
Settl. _den.	0.6205**	0.3091	-0.0359	0.1230	0.1178	0.1080
Comp. _den.	-0.6328**	0.2965	-	-	-	-
Comp. _den.^2	-0.1029	0.2961	-	-	-	-
Wear_den.Settl. _den.	-	-	-	-	0.2189**	0.0929
Wear_den.Comp. _den.	0.3396	0.2479	-	-	-	-
Mean VIF	15.58		3.01		3.06	
R^2	0.22		0.14		0.16	
Adj. R^2	0.18		0.12		0.14	

We transform all data by dividing by the sample median prior to taking logs. In that way, the first order coefficients can be interpreted as cost elasticities at the sample median. See Table 11 in appendix for definitions of the variables.

Note: ***, **, *: Significance at 1 %, 5 %, and 10 % level, respectively

Leaving out cost drivers that are correlated with the damage measures may lead to undesirable omitted variable bias. If that is the case, the coefficients in Model 1 are over- or underestimated. Hence, in Model 2, we include a set of control variables that we believe to be important in this context. The average quality class on a track section (Qual_ave, where low values indicate high speeds allowed) can be important to include as higher speeds imply stricter requirements on track quality (track geometry). This may increase the propensity to rectify the settlement damage caused by the vehicles. Indeed, the interaction term between Settlement and Qual_ave is negative, which suggests that the cost impact of settlement is lower for low linespeeds compared to high linespeeds. The first order coefficient for Qual_ave is negative, yet not significant. Here it should be noted that differences in track irregularities, curvature, linespeeds and traffic volume have been (at least substantially) normalized, as these aspects are inputs in the simulations and therefore picked up by the damage measures.

Rail age is also included in the model estimation. Older rails seem to be more costly. Considering that rail age is a proxy for track standard due to accumulated use (yet, not a perfect proxy), a positive and significant coefficient is intuitive as high maintenance costs on old and heavily used track is expected, which eventually makes a renewal economically justified.

Table 8: Estimation results, Model 2

	Model 2a		*Model 2b*	
	Coef.	Rob. Std. Err.	Coef.	Rob. Std. Err.
Cons.	16.4675***	0.1030	16.4779***	0.1024
Wear_den.	0.1079	0.0718	0.1182*	0.0714
RCF_den.	0.0485	0.0805	-	-
Settl._den.	0.0996	0.0983	0.1345*	0.0719
Track_length	0.9303***	0.0582	0.9385***	0.0588
Qual_ave	-0.0428	0.2185	-0.0237	0.2113
Qual_ave^2	-0.9850*	0.5132	-1.0099*	0.5205
Rail_age	0.2575*	0.1337	0.2699**	0.1340
Settl._den.Qual_ave	-0.5618***	0.1189	-0.5685***	0.1188
Region_West	0.3264**	0.1429	0.3254**	0.1431
Region_North	0.0442	0.1903	0.0395	0.1886
Region _Central	-0.2981**	0.1500	-0.2933*	0.1491
Region _South	-0.2179	0.1395	-0.2173	0.1395
Mean VIF	2.73		2.31	
R^2	0.70		0.70	
Adj. R^2	0.67		0.67	

We transform all data by dividing by the sample median prior to taking logs. In that way, the first order coefficients can be interpreted as cost elasticities at the sample median. See Table 11 in appendix for definitions of the variables.

Note: ***, **, *: Significance at 1 %, 5 %, and 10 % level, respectively

Turning to the cost elasticities with respect to the damage measures in Model 2a, we note that these are 0.1079, 0.0485 and 0.0996 for wear, RCF and settlement, respectively. None of these estimates are statistically significant. In Model 2b we drop RCF due to its high correlation coefficient with settlement, which generates a slightly higher estimate for settlement. The coefficients for wear and settlement are now statistically significant at the 10 per cent level. The sum of the first order coefficients is 0.2560 and 0.2527 in Model 2a and Model 2b, respectively, indicating that the cost impact of RCF is to a large extent picked up by the estimates for wear and settlement.

Settlement has a significant interaction term with Qualave. To produce an estimate for settlement with respect to the observed values of the track quality classification, we calculate the cost elasticity at the sample mean, based on equation (6).

$$\hat{\gamma}_{iSettl.} = \hat{\beta}_1 + 2 \cdot \hat{\beta}_2 lnQualave_i, \quad (6)$$

This results in a cost elasticity at 0.1910 (p-value=0.010).

6.1 Marginal costs

To calculate the marginal costs of different vehicle types, we first need to estimate the marginal cost of each damage mechanism. These costs are then linked to vehicle types based on the amount of damage per ton-km each vehicle has caused according to the simulations in the first stage of our estimation approach. In that way, we produce a marginal cost per ton-km which is the preferred charging unit. However, as shown by the estimation results in the previous section, there is a strong correlation between the different damages, making it difficult to isolate their relative cost impacts. Given that settlement is strongly correlated with track component fatigue and RCF, we expect the estimate of settlement to include the effect of the two latter damages to a large extent, which is corroborated by the estimation results in Model 2b. This generated damage estimates that are statistically significant at the 10 per cent level.

In the marginal cost estimation presented below, we use the estimated cost elasticities for wear and settlement (evaluated at the sample median). Marginal costs that are based on the non-significant cost elasticities in Model 2a are presented in Table 13 in appendix.

The marginal cost of a damage mechanism j is formulated as

$$MC_j = \frac{\partial C}{\partial D_j} = \frac{D_j}{C}\frac{\partial C}{\partial D_j}\frac{C}{D_j} = \frac{\partial lnC}{\partial lnD_j}\frac{C}{D_j}, \quad (7)$$

where D is damage. Hence, from equation (7) we can be express the marginal cost estimate as

$$MC_j = \hat{\gamma}_j \cdot \widehat{AC}_j, \quad (8)$$

where $\hat{\gamma}_j$ is the estimated cost elasticity of damage mechanism j. $\widehat{AC}_j$ is the average cost $(\frac{\hat{C}}{D_j})$, where $\hat{C}$ is predicted costs specified as

$$\hat{C}_i = \exp[\ln(C_i) - \hat{v}_i + 0.5\hat{\sigma}^2], \quad (9)$$

Equation (9) derives from the double-log specification and the assumption of normally distributed residuals (see for example Munduch et al. 2002).

We use a weighted marginal cost for the 143 track sections in this study, according to equation (10) below. This implies that we use the damage share of each section, which produces a marginal cost estimate that generates the same income - when applied to the vehicle's damage types per ton-km - as if each section's marginal cost would be used.

$$MC_{ij}^{W} = MC_{ij} \cdot \frac{D_{ij}}{(\sum_i D_{ij})/N}, \quad (10)$$

The average cost and the weighted marginal costs are presented in Table 9.[4] These costs become quite low as they are estimates per total damage.

Table 9: Average and marginal costs per damage unit, SEK in 2014 prices

	Variable	Mean	Std. Err.	[95% Conf.	Interval]
Average cost	Wear	5.62E-03	7.90E-04	4.05E-03	7.18E-03
	Settlement	2.56E-07	5.08E-08	1.56E-07	3.57E-07
Marginal cost	Wear	6.64E-04	9.34E-05	4.79E-04	8.48E-04
	Settlement	3.45E-08	6.84E-09	2.10E-08	4.80E-08
Weighted marginal cost	Wear	1.41E-04	8.97E-06	1.23E-04	1.59E-04
	Settlement	9.37E-09	5.97E-10	8.19E-09	1.05E-08

The marginal cost for settlement is lower than the cost for wear, even though their respective cost elasticities are similar. The reason is that the damages have different units, generating an average cost of settlement that is much lower than the average cost of wear. However, estimates that are comparable with the costs in the literature on marginal rail infrastructure costs in Sweden are produced when multiplying the estimates in Table 9 with the damage caused by a ton-km of a certain vehicle (marginal rail infrastructure costs are expressed as marginal cost per ton-km in the literature). The resulting marginal costs per vehicle and damage type presented in Table 10 do not differ at the same order of magnitude as the costs in Table 9. In other words, the differences in units between the damage mechanisms are normalized in Table 10.

Before we estimate a marginal cost per vehicle type, we examine the average marginal costs for all vehicles in order to make a comparison with previous estimates on Swedish data. More specifically, we use the mean wear per ton-km and mean settlement per ton-km for all traffic on the 143 sections and multiply with the marginal cost as well as the weighted marginal costs presented in Table 9. The marginal costs per ton-km for wear and settlement

[4] Note that the mean of the weighted marginal cost in equation (10) is the same value as the sum of the weighted marginal costs ($MC^W = \sum_{ij}\left[MC_{ij} \cdot \frac{D_{ij}}{(\sum_i D_{ij})}\right]$) as specified in Munduch et al. (2002) and Andersson (2008).

are illustrated in Figure 6, showing that costs fall sharply with ton density. Similar shapes were found for a number of European countries (including Sweden) in Wheat et al. (2009). The weighted marginal cost per ton-km is 0.0148 SEK, which is the sum of both damage mechanisms' weighted marginal cost. Using a weighted average of the vehicle's damages per ton-km, we get a marginal cost at 0.0131 SEK.[5] These estimates are higher than previous estimates on Swedish data in Andersson (2008) and Odolinski and Nilsson (2015), which are 0.0081 SEK and 0.0083 SEK respectively (in 2013 prices).

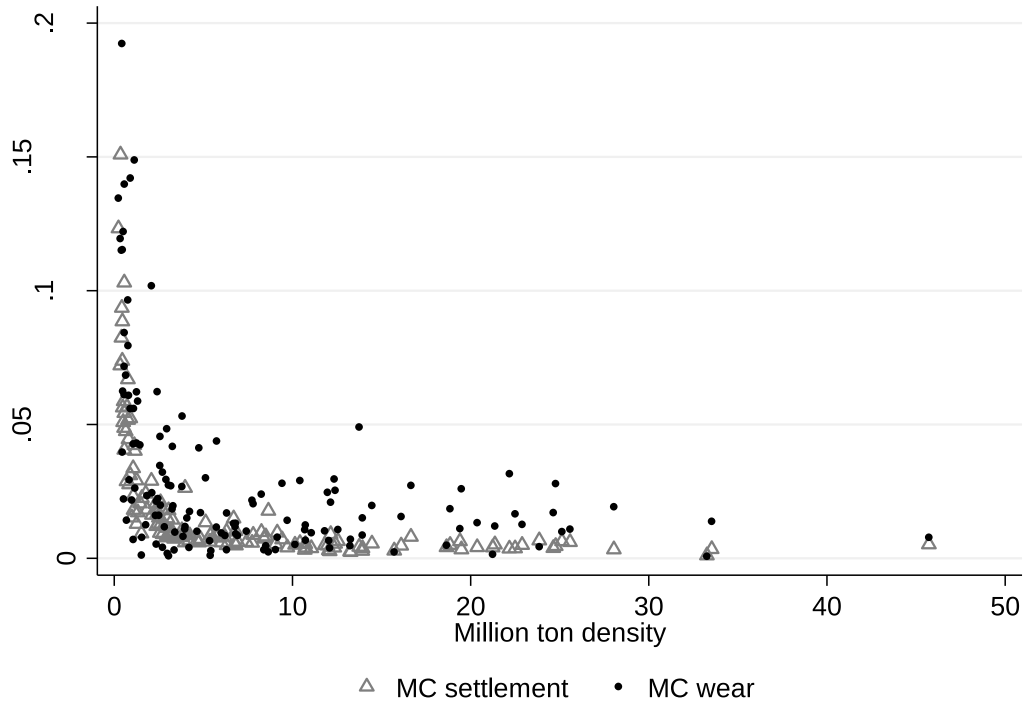

Figure 6: Marginal costs for settlement and wear

Similar to Smith et al. (2014), we use the simulation results to calculate a marginal cost per ton-km and vehicle type. Specifically, we have run simulations for the different vehicles on the 143 track sections (as described in section 3), producing a damage value per ton-km for a

[5] The share of vehicle types' ton-km on a section with respect to their total ton-km is used as weights.

vehicle type on a certain track section. We use a mean value of the damage per ton-km a vehicle has incurred on the sections it ran on during 2014.[6] The product of a vehicle's mean value (damage per ton-km) and the marginal cost of the corresponding damage mechanism is the marginal cost per ton-km for that vehicle. For example, freight loco 4x20 t, V_{max} 120 km/h, has a mean wear per ton-km at 32.4 and a mean settlement per ton-km at 743 564. Its marginal cost is therefore 32.4*1.41-E04 + 743 564*9.37E-09 = 0.0115 SEK.

The damage per ton-km for each vehicle type is presented in Table 10. Note that these values partly depend on which track sections the different vehicles ran on during 2014, considering that track quality differs between the sections. More specifically, we used measurements on track geometry and track irregularities as input in the simulation, as this will affect the damage caused by traffic. Hence, considering that each vehicle type did not run on all the 143 track sections, the values in Table 10 are not completely normalized. The damage measures for RCF and track component fatigue are presented in Table 13 and Table 15 in appendix.

The calculated marginal costs for the different vehicle categories are presented in Table 10 along with their damage measures. The vehicles are ordered after the highest marginal cost, showing that Motor coach 4x21 t, V_{max} 200 km/h (stiff wheelset guidance and high center of gravity) is assigned a marginal cost almost twice as large as the cost for the vehicle type with the second highest estimate. The other estimates stretch from 0.0096 SEK to 0.0186 SEK, indicating rather differentiated marginal costs.

Interestingly, a laden freight wagon 2x22t, V_{max} 100km, has the lowest marginal cost (0.0096 SEK), while its tare counterpart has a marginal cost at 0.0132 SEK. The reason for this relationship is that the tare freight wagon has a factor 1.81 higher wear per ton-km than a laden freight wagon, while the laden wagon only has a factor 1.17 higher settlement per ton-

[6] We prefer the mean values to the weighted averages as the latter measure is even more dependent on which track sections the vehicles have been running on during 2014.

km than the tare wagon (cf. Table 10). Considering that the cost elasticities for the different damage types are rather similar, these differences in damages are reflected in the marginal costs.

Table 10: Marginal costs per ton-km and vehicle type, SEK in 2014 prices

Vehicle type	Mean wear per ton-km	Mean settlement per ton-km	Marginal cost per ton-km, SEK
Motor coach 4x21 t, V_{max} 200 km/h, high center of gravity**	207.6	1 008 441	0.0387
Three-piece bogie 4x30 t, V_{max} 60 km/h	87.1	867 067	0.0204
Passenger car 4x14 t, V_{max} 160 km/h	81.5	756 207	0.0186
Motor coach with Jacob bogie 3x16.5 t, V_{max} 160 km/h**	93.9	502 314	0.0179
Freight loco 6x20 t, V_{max} 120 km/h	60.3	967 401	0.0176
Passenger Loco 4x19 t, V_{max} 140 km/h	73.0	762 578	0.0174
Y25 bogie 4x22 t, V_{max} 100 km/h	58.3	815 878	0.0158
Passenger Loco 4x19 t, V_{max} 175 km/h	60.2	770 689	0.0157
Motor coach 4x16 t, V_{max} 200 km/h**	40.8	877 980	0.0140
Freight wagon 2x6.5, V_{max} 100 km/h	67.2	394 093	0.0132
Freight loco 6x30 t, V_{max} 70 km/h	21.8	995 613	0.0124
Motor coach 4x16 t, V_{max} 200 km/h*	36.8	696 335	0.0117
Freight loco 4x20 t, V_{max} 120 km/h	32.4	743 564	0.0115
Motor coach with Jacob bogie 3x12.5 t, V_{max} 200 km/h*	43.8	571 067	0.0115
Motor coach 4x12 t, V_{max} 140 km/h*	33.9	682 236	0.0112
Three-piece bogie 4x6.5 t, V_{max} 60 km/h	32.1	603 891	0.0102
Freight wagon 2x22 t, V_{max} 100 km/h	37.1	462 377	0.0096

* Flexible wheelset guidance, ** Stiff wheelset guidance

Finally, it should be pointed out that the cost elasticities we use in the marginal cost estimation are considered to also capture effects of RCF and track component fatigue. However, the correlation between the vehicle’s damages per ton-km is quite low compared to the correlation coefficients in Table 6 that are calculated for track sections. For example, the correlation coefficient between wear and RCF for the different vehicles is 0.10, and 0.54 between settlement and track component fatigue. This implies that the relationship between the vehicles in Table 10 would be different if we had been able to isolate the relative costs of all damage mechanisms. Hence, these estimates should be interpreted with care.

The marginal costs from Model 2a (presented in Table 13 in appendix) include the cost impact from RCF which generates a slightly different relationship between the vehicles' costs. Still, these costs are based on cost elasticities with respect to damages that are not statistically significant.

7.0 Conclusion

This paper contributes to the existing literature by showing that the two-stage method in Smith et al. (2014) can produce estimates on the relative cost of damage mechanisms that can be informative for infrastructure managers in Europe. Specifically, by combining engineering and econometric approaches, we have estimated marginal costs for the vehicle types running on the Swedish railway network. We have developed previous work on this method by using a larger set of - and more detailed - vehicle models, as well as a larger set of track sections that constitutes a major part of the Swedish railway network.

The different damage mechanisms proved to be highly correlated between track sections, making it difficult to isolate the cost impact of each damage type. Still, our model was able to provide significant cost elasticities with respect to wear and settlement that could be used in the estimation of marginal costs. The estimates for these two damage mechanisms capture the cost impact from RCF and track component fatigue to a large extent. However, the downside is that the resulting marginal costs do not reflect the relative differences in RCF per ton-km and track component fatigue per ton-km between the vehicle types.

The results in this paper indicate a substantial variation in the marginal cost per ton-km for different vehicle types running on the Swedish railway, which is due to differences in the damage done by the vehicles and the relative cost of the damage mechanisms. Track access charges with similar relative differences between vehicle types would create strong incentives for using more track friendly vehicles.

More observations over time can be valuable for future research in order to generate more reliable and robust estimates. The results from our approach can also be used to differentiate the track access charges with respect to, for example, line speed, in line with the charges that Switzerland has proposed to implement in 2017. More specifically, future work can use the simulation results on how line speed adds to different damages. Together with the relative costs of these damage mechanisms, it is then possible to calculate marginal costs for different vehicles that are also differentiated with respect to line speeds.

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Appendix

Table 11: Definition of variables

Wear_den.	=	ln(wear density)
Wear_den.^2	=	ln(wear density)*ln(wear density)
RCF_den	=	ln(RCF density)
Settl._den	=	ln(Settlement density)
Comp._den,	=	ln(Track component fatigue density)
Comp._den.^2	=	ln(Track component fatigue density)*ln(Track component fatigue density)
Wear_den.Settl._den	=	ln(wear density)*ln(settlement density)
Wear_den.Com._den.	=	ln(wear density)*ln(Track component fatigue density)
Track_length	=	ln(track length)
Qual_ave	=	ln(average quality class)
Qual_ave^2	=	ln(average quality class)* ln(average quality class)
Rail_age	=	ln(average age of rails)
Settl._den.Qual_ave	=	ln(settlement density)*ln(average quality class)
Region_West	=	Dummy for region West
Region_North	=	Dummy for region North
Region_Central	=	Dummy for region Central
Region_South	=	Dummy for region South
Region_East	=	Dummy for region East

Table 12: Model 2a - Average and marginal costs per damage unit, SEK in 2014 prices (based on non-statistically significant cost elasticities)

	Variable	Mean	Std. Err.	[95% Conf.	Interval]
Average cost	Wear	5.60E-03	7.84E-04	4.05E-03	7.15E-03
	RCF	1.65E+00	2.12E-01	1.23E+00	2.07E+00
	Settlement	2.58E-07	5.27E-08	1.54E-07	3.63E-07
Marginal cost	Wear	6.04E-04	8.46E-05	4.37E-04	7.72E-04
	RCF	8.02E-02	1.03E-02	5.99E-02	1.01E-01
	Settlement	2.57E-08	5.25E-09	1.53E-08	3.61E-08
Weighted marginal cost	Wear	1.29E-04	8.21E-06	1.13E-04	1.45E-04
	RCF	2.34E-02	1.49E-03	2.04E-02	2.63E-02
	Settlement	6.94E-09	4.43E-10	6.07E-09	7.82E-09

Table 13: Model 2a - Marginal costs per ton-km and vehicle type, SEK in 2014 prices (based on non-statistically significant cost elasticities)

Vehicle type	Mean wear per ton-km	Mean settlement per ton-km	Mean RCF per ton-km	Marginal cost per ton-km, SEK
Motor coach 4x21 t, V_{max} 200 km/h, high center of gravity**	207.6	1 008 441	0.08	0.0356
Three-piece bogie 4x30 t, V_{max} 60 km/h	87.1	867 067	0.25	0.0231
Freight wagon 2x6.5, V_{max} 100 km/h	67.2	394 093	0.37	0.0200
Passenger car 4x14 t, V_{max} 160 km/h	81.5	756 207	0.16	0.0195
Motor coach with Jacob bogie 3x16.5 t, V_{max} 160 km/h**	93.9	502 314	0.11	0.0181
Freight loco 6x20 t, V_{max} 120 km/h	60.3	967 401	0.10	0.0167
Passenger Loco 4x19 t, V_{max} 140 km/h	73.0	762 578	0.07	0.0164
Y25 bogie 4x22 t, V_{max} 100 km/h	58.3	815 878	0.10	0.0155
Passenger Loco 4x19 t, V_{max} 175 km/h	60.2	770 689	0.10	0.0154
Motor coach 4x16 t, V_{max} 200 km/h**	40.8	877 980	0.14	0.0146
Motor coach with Jacob bogie 3x12.5 t, V_{max} 200 km/h*	43.8	571 067	0.17	0.0135
Motor coach 4x16 t, V_{max} 200 km/h*	36.8	696 335	0.10	0.0118
Freight wagon 2x22 t, V_{max} 100 km/h	37.1	462 377	0.12	0.0108
Freight loco 4x20 t, V_{max} 120 km/h	32.4	743 564	0.06	0.0108
Freight loco 6x30 t, V_{max} 70 km/h	21.8	995 613	0.04	0.0107
Three-piece bogie 4x6.5 t, V_{max} 60 km/h	32.1	603 891	0.09	0.0105
Motor coach 4x12 t, V_{max} 140 km/h*	33.9	682 236	0.06	0.0104

* Flexible wheelset guidance, ** Stiff wheelset guidance

Table 14: Descriptive statistics, obs. from 169 track sections

	Median	Mean	St. dev.	Min	Max
Maintenance costs, million SEK	13.71	19.71	22.42	0.53	209.22
Track length, km	58.71	72.51	51.85	4.52	251.39
Average quality class	2.89	2.94	1.15	1.00	5.17
Average age of rails	21.2	22.3	9.6	2.3	53.1
Million ton density	4.52	8.15	9.39	0.00	61.98
Region West	0	0.20	0.40	0	1
Region North	0	0.14	0.34	0	1
Region Central	0	0.18	0.39	0	1
Region South	0	0.25	0.43	0	1
Region East	0	0.24	0.43	0	1